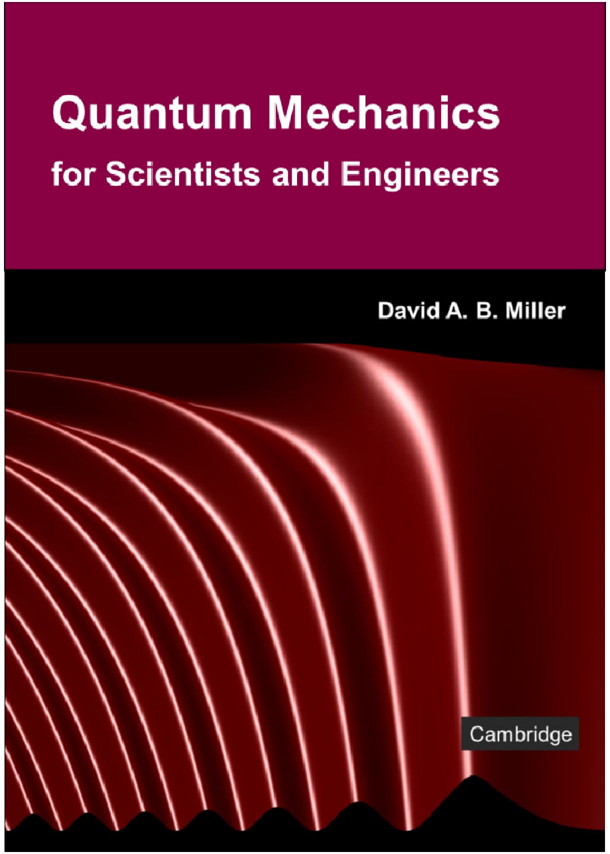
2 Classical mechanics, oscillations and waves

Slides: Lecture 2a Useful ideas from classical physics

Text reference: Quantum Mechanics for Scientists and Engineers

Section Appendix B



Classical mechanics, oscillations and waves

Useful ideas from classical physics

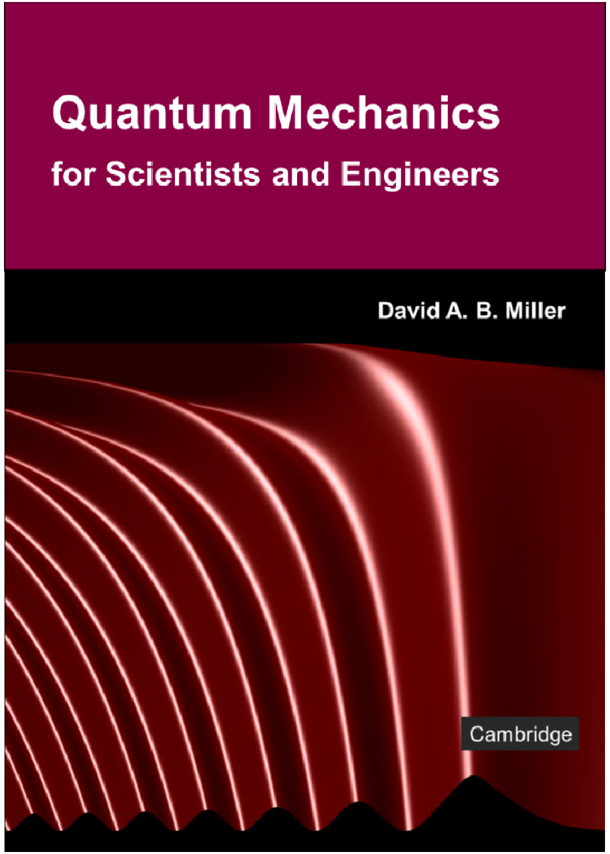
David Miller

2 Classical mechanics, oscillations and waves

Slides: Lecture 2b Elementary classical mechanics

Text reference: Quantum Mechanics for Scientists and Engineers

Section B.1



Classical mechanics, oscillations and waves

Elementary classical mechanics

Quantum mechanics for scientists and engineers

David Miller

Momentum and kinetic energy

For a particle of mass m

- the classical momentum
  - which is a vector
    - because it has direction
      - is $\mathbf{p} = m\mathbf{v}$
      - where $\mathbf{v}$ is the (vector) velocity

The kinetic energy

- the energy associated with motion
  - is

$$K.E. = \frac{p^2}{2m}$$

Momentum and kinetic energy

In the kinetic energy expression

$$K.E. = \frac{p^2}{2m}$$

we mean

$$p^2 \equiv \mathbf{p} \cdot \mathbf{p}$$

i.e.,

the vector dot product of $\mathbf{p}$ with itself

Potential energy

Potential energy is defined as

energy due to position

It is usually denoted by V in quantum mechanics

even though this potential energy

in units of Joules

might be confused with the idea of voltage

in units of Joules/Coulomb

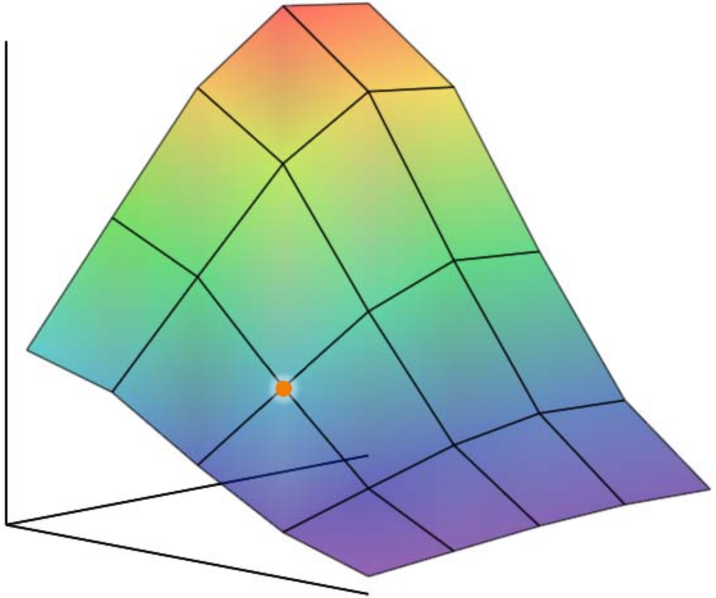
and even though we will use voltage

often in quantum mechanics

Potential energy

Since it is energy due to position

- it can be written as $V(\mathbf{r})$
  - We can talk about potential energy
    - if that energy only depends on where we are
      - not how we got there



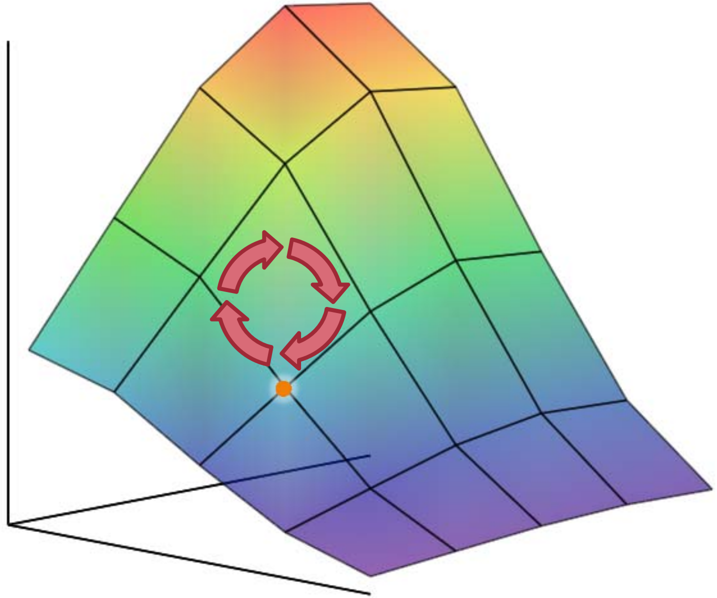
Potential energy

Classical “fields” with this property are called

- “conservative” or “irrotational”
  - the change in potential energy round any closed path is zero

Not all fields are conservative

- e.g., going round a vortex
- but many are conservative
  - gravitational, electrostatic



The Hamiltonian

The total energy can be

the sum of the potential and kinetic energies

When this total energy is written as a function of position and momentum

it can be called the (classical) "Hamiltonian"

For a classical particle of mass m in a conservative potential $V(\mathbf{r})$

$$H = \frac{p^2}{2m} + V(\mathbf{r})$$

Force

In classical mechanics

we often use the concept of force

Newton's second law relates force and acceleration

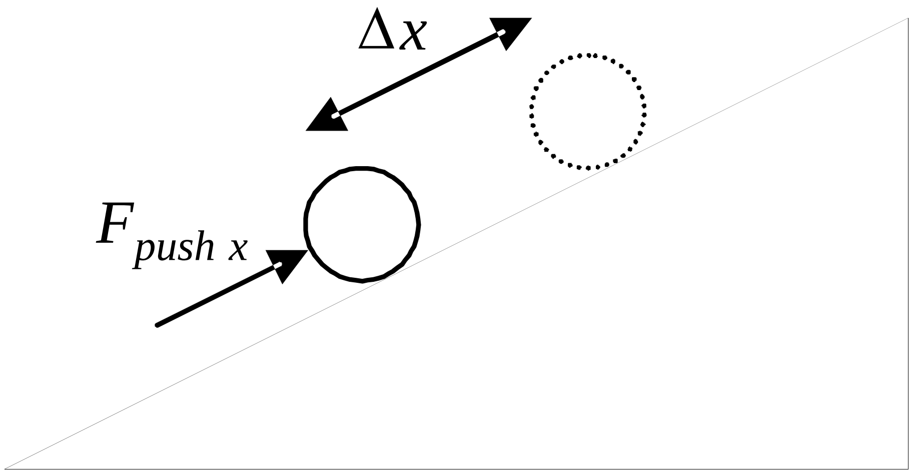
$$\mathbf{F} = m\mathbf{a}$$

where m is the mass and $\mathbf{a}$ is the acceleration

Equivalently

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

where $\mathbf{p}$ is the momentum



Force and potential energy

We get a change ΔV in potential energy V

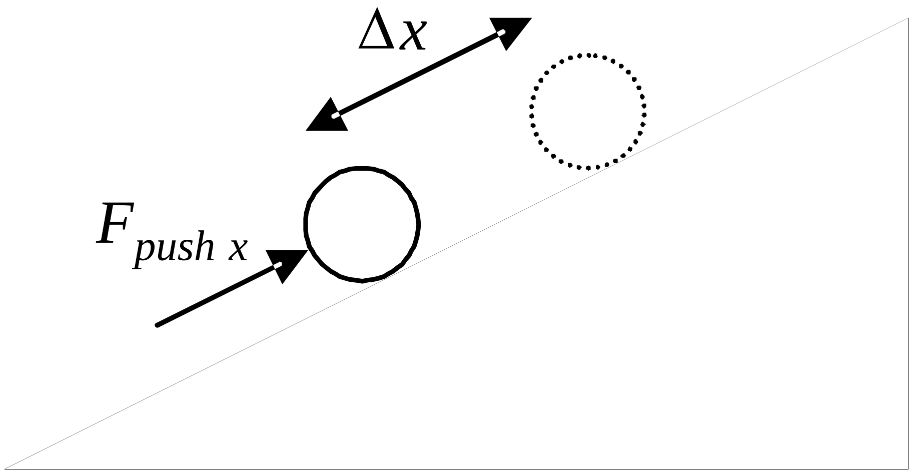
$$\Delta V = F_{pushx} \Delta x$$

by exerting a force F_{pushx}

in the x direction up the slope

through a distance Δx

Force and potential energy



Equivalently $$F_{pushx} = \frac{\Delta V}{\Delta x}$$

or in the limit $$F_{pushx} = \frac{dV}{dx}$$

The force exerted by the potential gradient on the ball is downhill

so the relation between force and potential is

$$F_x = -\frac{dV}{dx}$$

Force as a vector

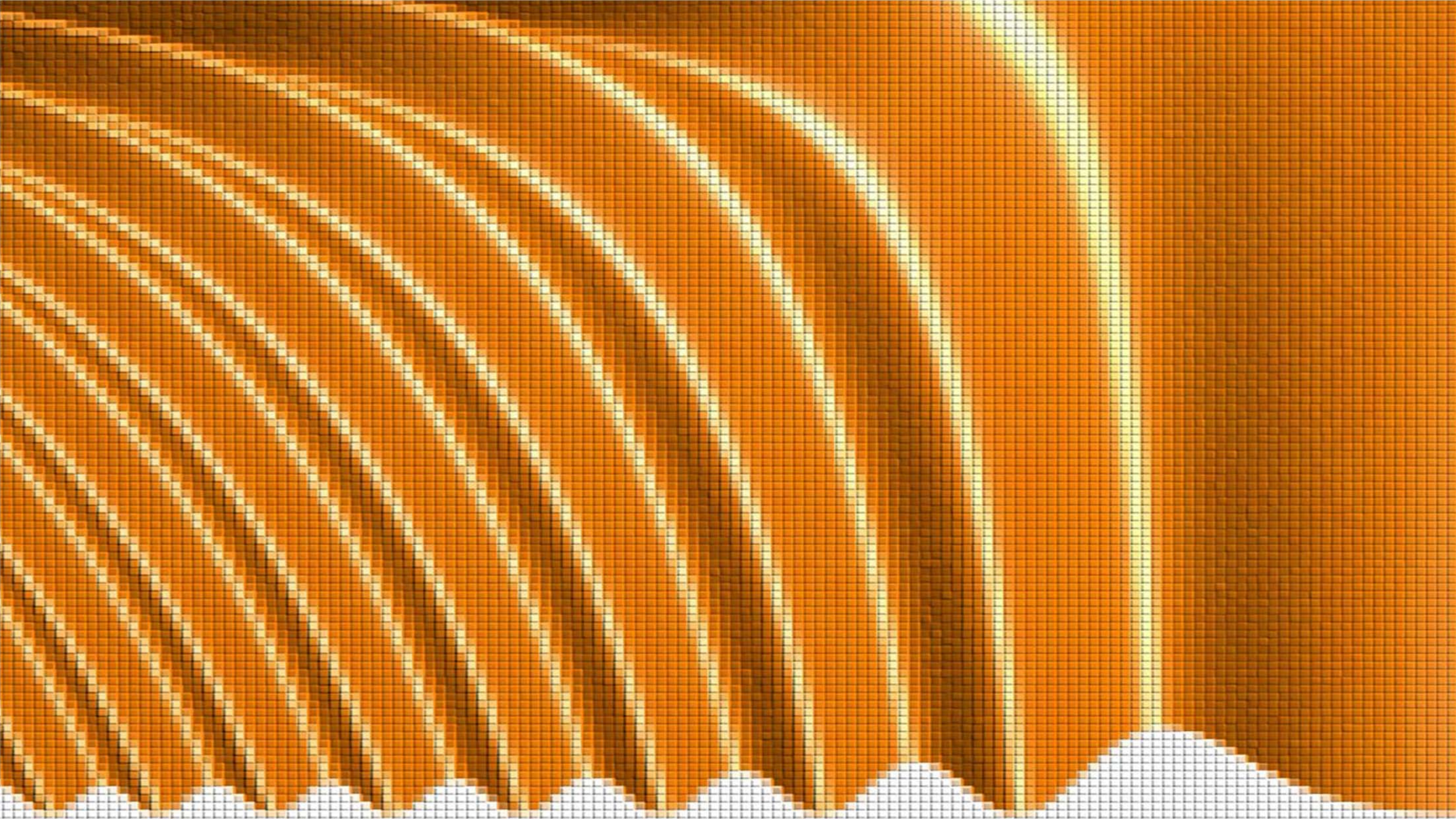
We can generalize the relation between potential and force

to three dimensions

with force as a vector

by using the gradient operator

$$\mathbf{F} = -\nabla V \equiv -\left[\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k} \right]$$

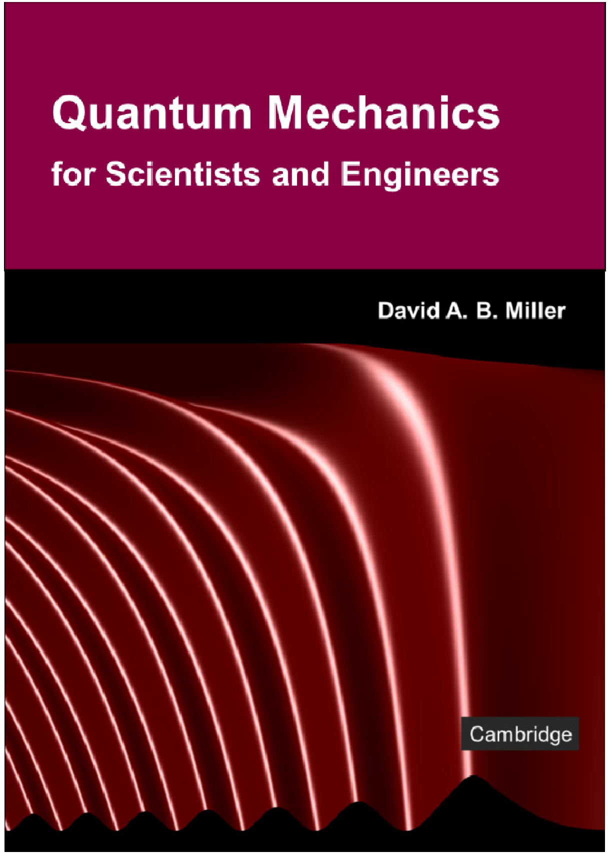


2 Classical mechanics, oscillations and waves

Slides: Lecture 2c Oscillations

Text reference: Quantum Mechanics for Scientists and Engineers

Section B.3



Classical mechanics, oscillations and waves

Oscillations

Quantum mechanics for scientists and engineers

David Miller

Mass on a spring

A simple spring will have a restoring force F acting on the mass M

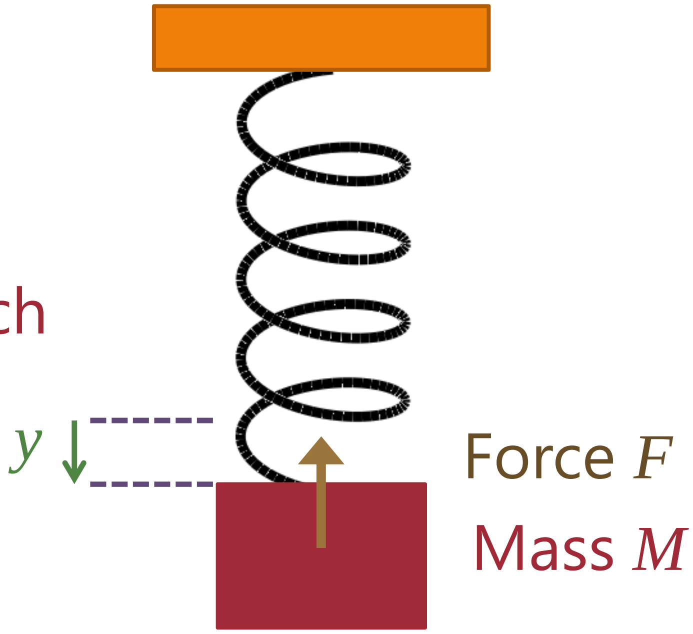
- proportional to the amount y by which it is stretched
  - For some "spring constant" K

$$F = -Ky$$

The minus sign is because this is "restoring"

- it is trying to pull y back towards zero

This gives a "simple harmonic oscillator"



Mass on a spring

From Newton's second law

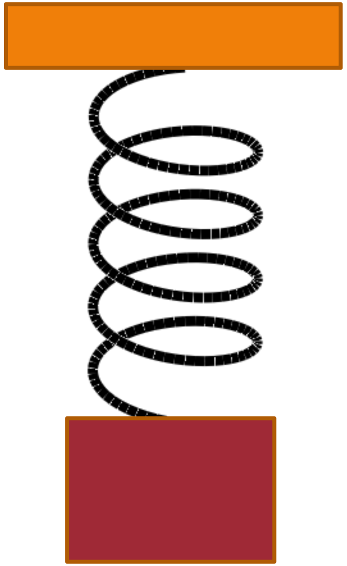
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$



angular frequency ω, in "radians/second" = $2\pi f$ where f is frequency in Hz

Mass on a spring

From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

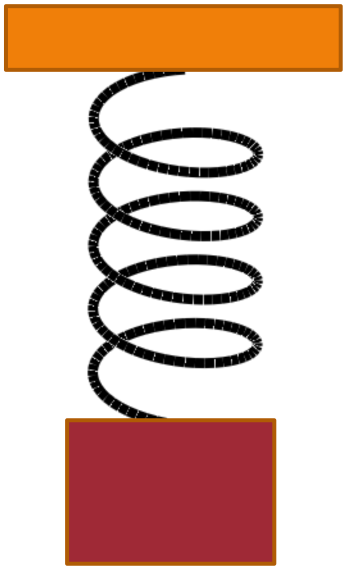
i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

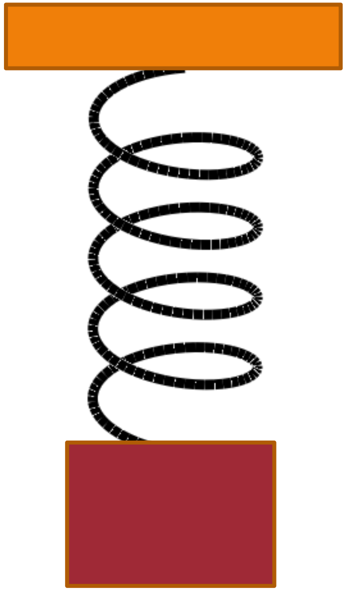
we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

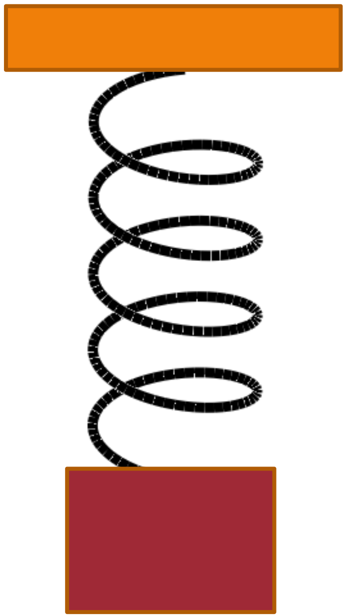
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$

Mass on a spring

From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

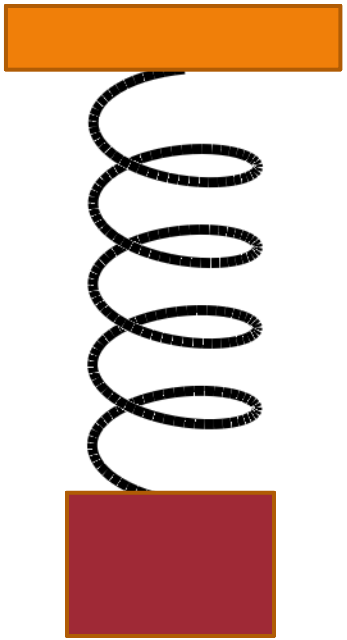
i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$



Mass on a spring

From Newton's second law

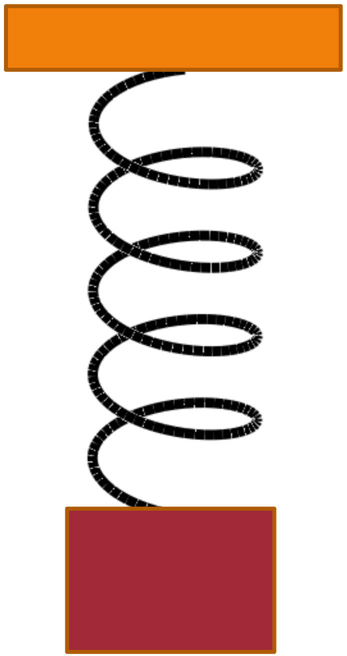
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K / M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K / M}$

e.g., $y \propto \sin \omega t$



Mass on a spring

From Newton's second law

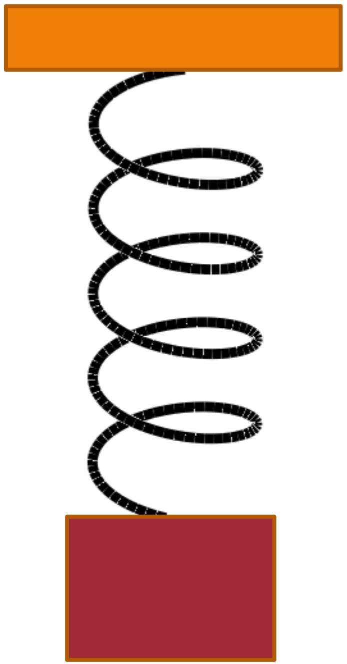
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$



Mass on a spring

From Newton's second law

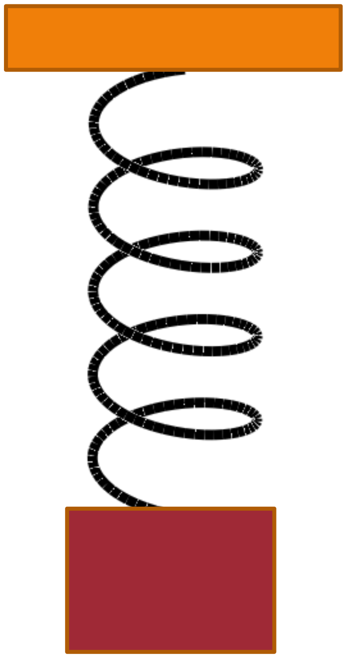
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin\omega t$



Mass on a spring

From Newton's second law

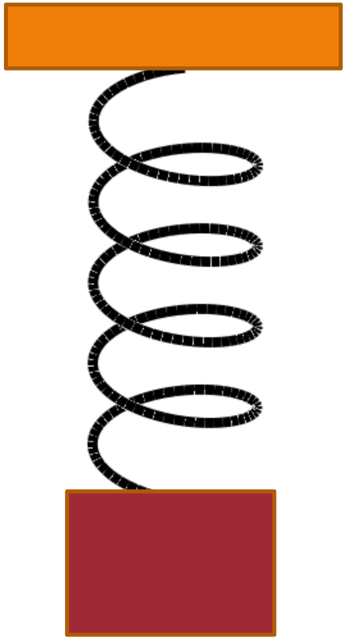
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$



Mass on a spring

From Newton's second law

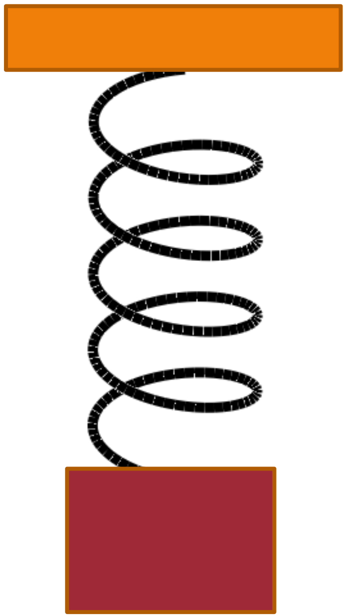
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

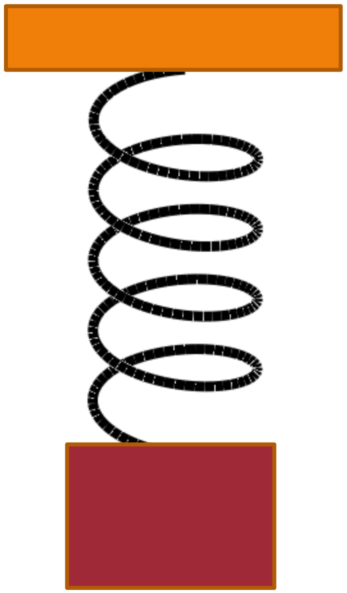
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring

From Newton's second law

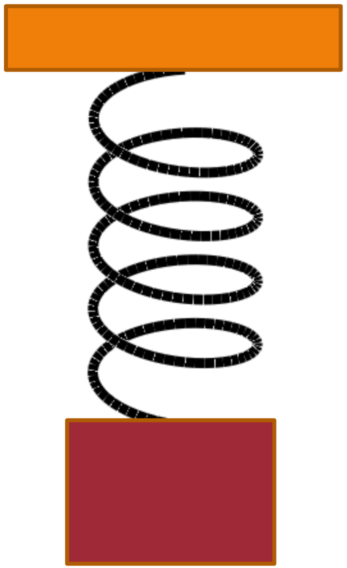
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

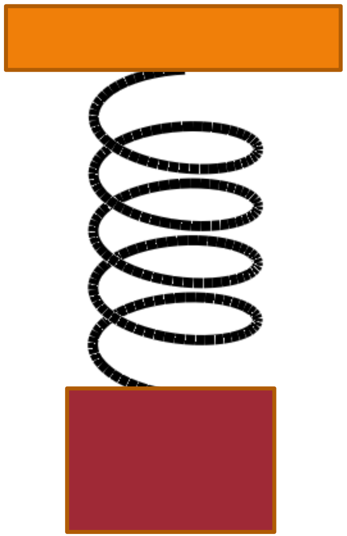
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $$y \propto \sin \omega t$$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2 y}{dt^2} = -Ky$$

i.e., $$\frac{d^2 y}{dt^2} = -\frac{K}{M} y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring

From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

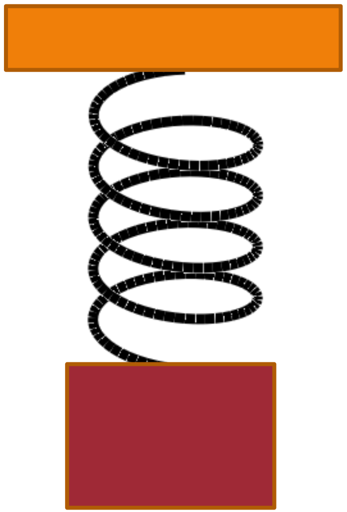
i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

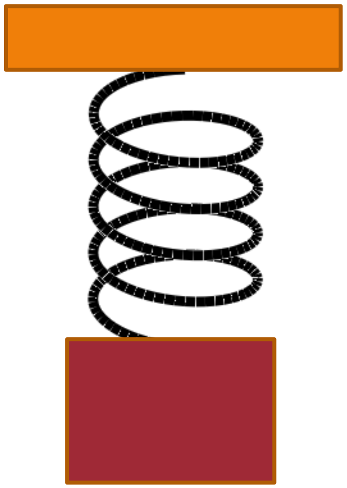
we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

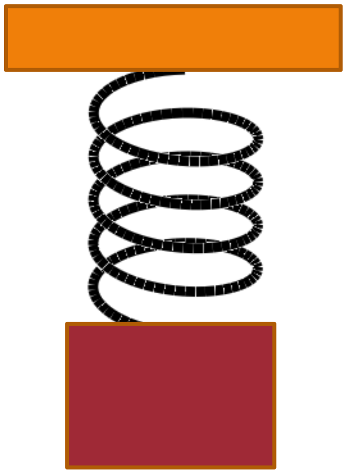
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

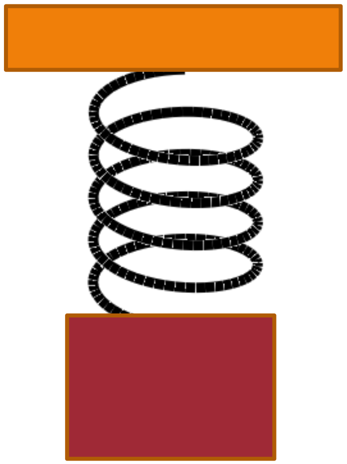
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring

From Newton's second law

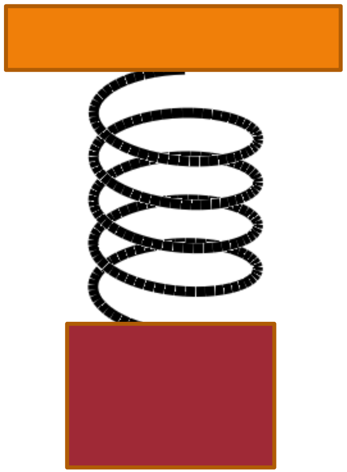
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

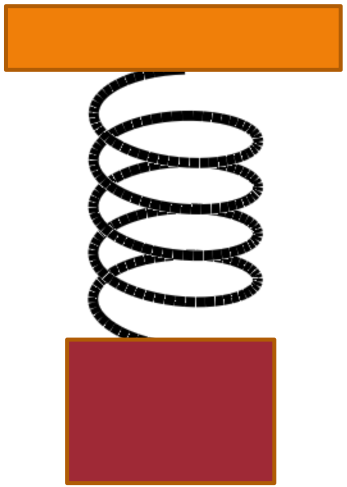
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

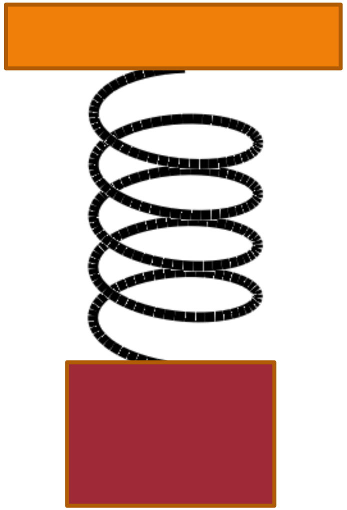
i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$

Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

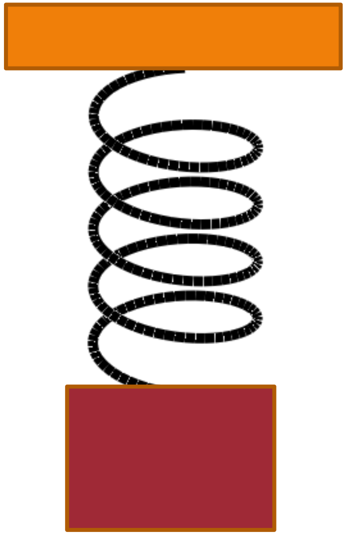
i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $y \propto \sin \omega t$

Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$

Mass on a spring

From Newton's second law

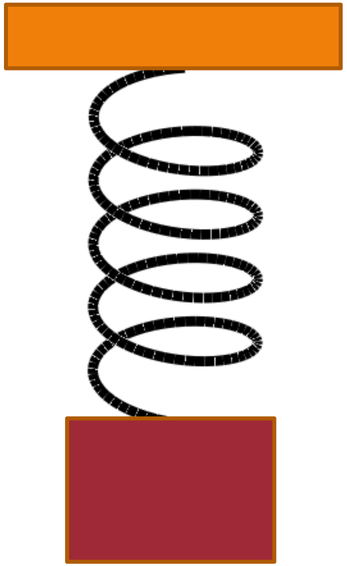
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

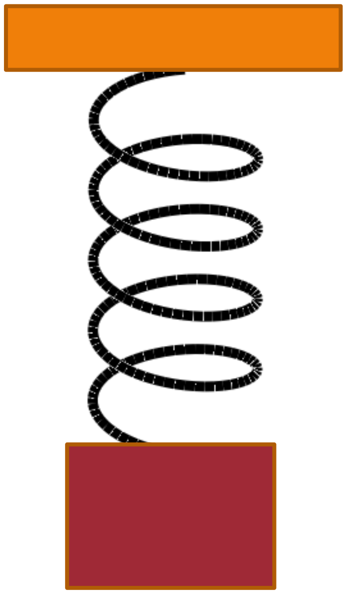
where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $$y \propto \sin \omega t$$



Mass on a spring



From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $$y \propto \sin \omega t$$

Mass on a spring

From Newton's second law

$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

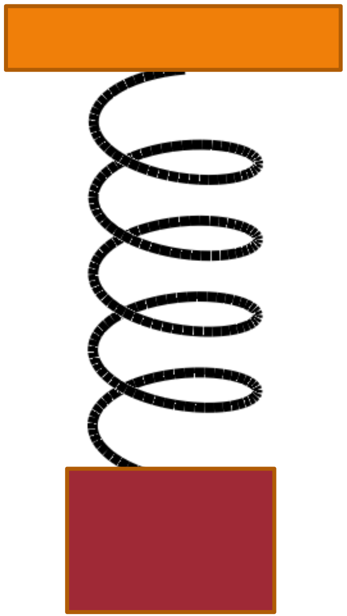
i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g.,

$$y \propto \sin \omega t$$



Simple harmonic oscillator

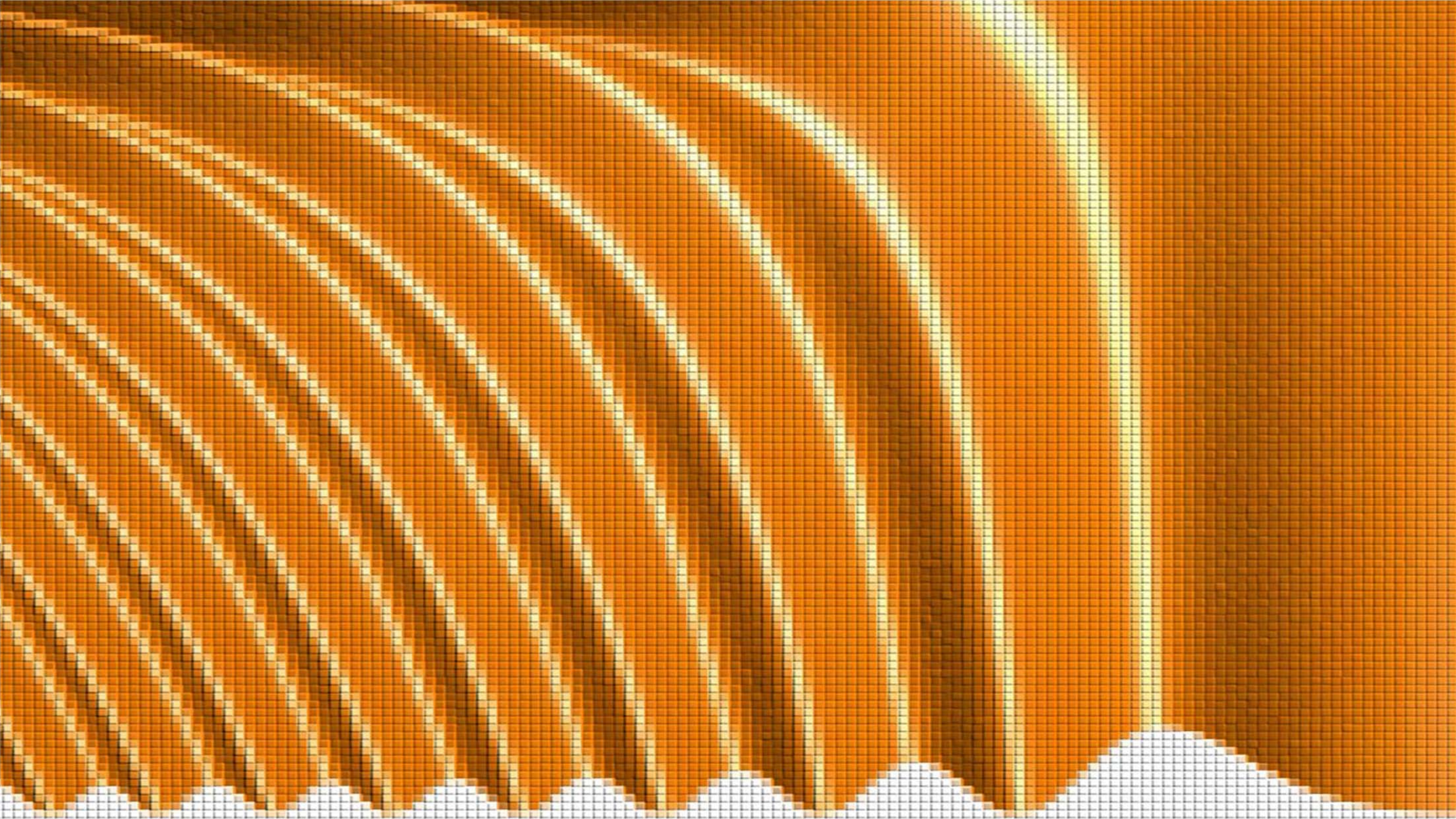
A physical system described by an equation like

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

is called a simple harmonic oscillator

Many examples exist

- mass on a spring
  - in many different forms
- electrical resonant circuits
- "Helmholtz" resonators in acoustics
- linear oscillators generally

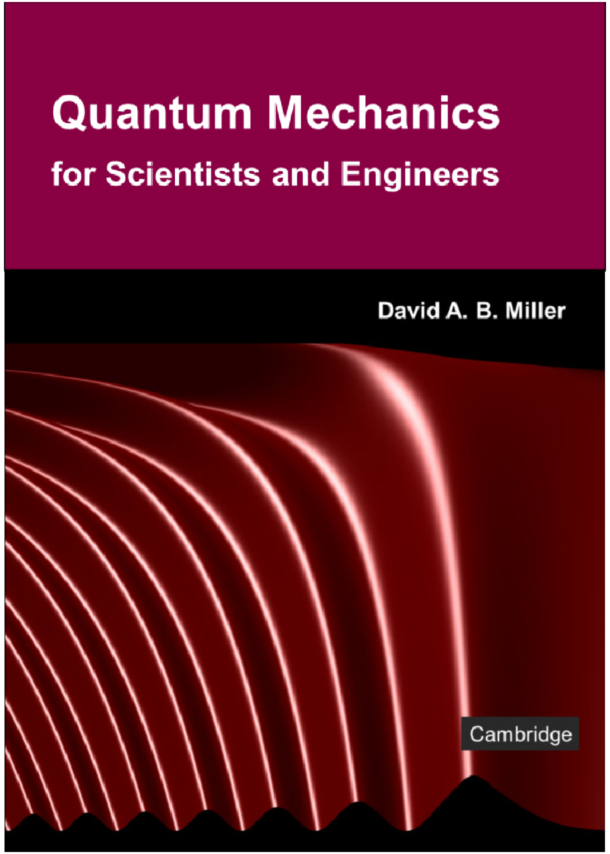


2 Classical mechanics, oscillations and waves

Slides: Lecture 2d The classical wave equation

Text reference: Quantum Mechanics for Scientists and Engineers

Section B.4



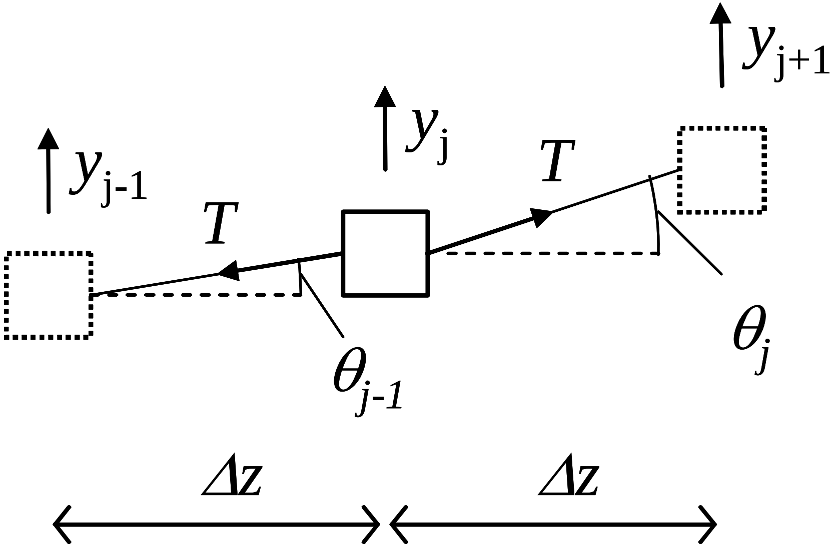
Classical mechanics, oscillations and waves

The classical wave equation

Quantum mechanics for scientists and engineers

David Miller

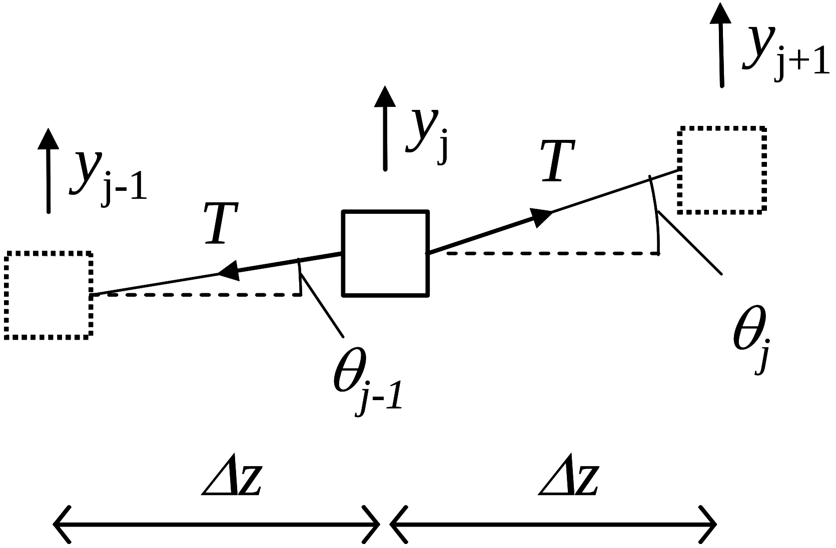
Classical wave equation



Imagine a set of identical masses connected by a string that is under a tension T

the masses have vertical displacements y_j

Classical wave equation



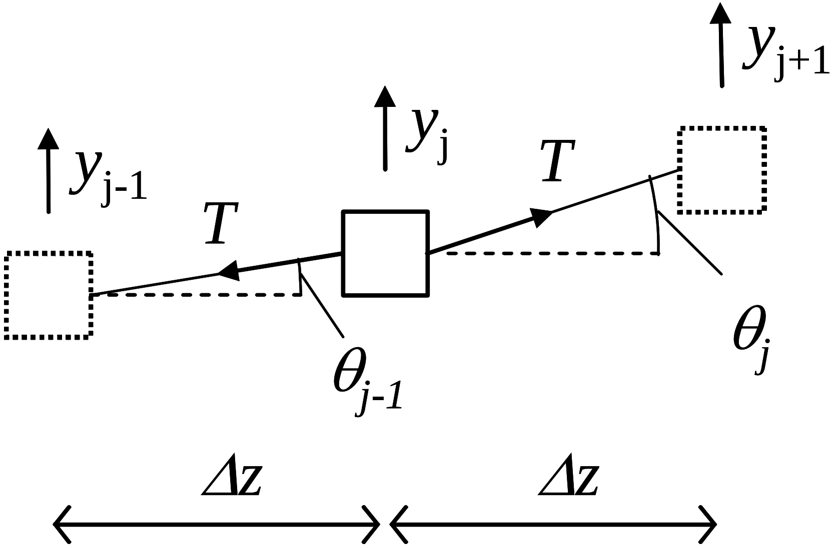
A force $T\sin\theta_j$ pulls mass j upwards

A force $T\sin\theta_{j-1}$ pulls mass j downwards

So the net upwards force on mass j is

$$F_j = T\left(\sin\theta_j - \sin\theta_{j-1}\right)$$

Classical wave equation



For small angles

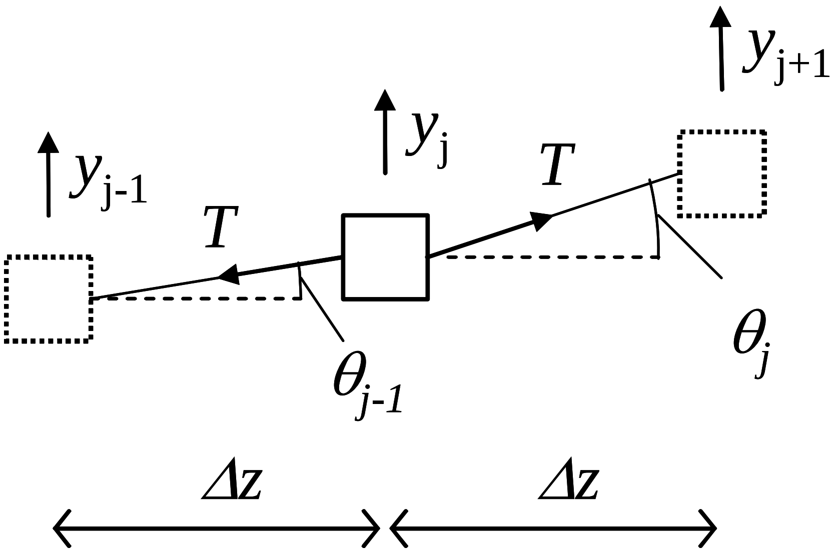
$$\sin\theta_j \simeq \frac{y_{j+1} - y_j}{\Delta z}, \quad \sin\theta_{j-1} \simeq \frac{y_j - y_{j-1}}{\Delta z}$$

So $F_j = T\left(\sin\theta_j - \sin\theta_{j-1}\right)$

becomes

$$F_j \simeq T\left[\frac{y_{j+1} - y_j}{\Delta z} - \left(\frac{y_j - y_{j-1}}{\Delta z}\right)\right]$$

$$= T\left[\frac{y_{j+1} - 2y_j + y_{j-1}}{\Delta z}\right]$$



Classical wave equation

In the limit of small Δz

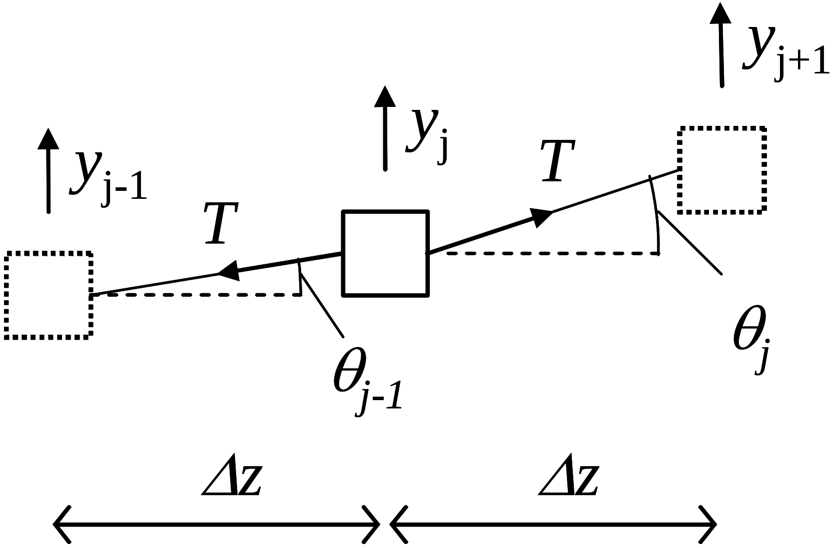
the force on the mass j is

$$F = T\left[\frac{y_{j+1} - 2y_j + y_{j-1}}{\Delta z}\right]$$

$$= T\Delta z\left[\frac{y_{j+1} - 2y_j + y_{j-1}}{(\Delta z)^2}\right]$$

$$= T\Delta z\frac{\partial^2 y}{\partial z^2}$$

Classical wave equation



Note that, with

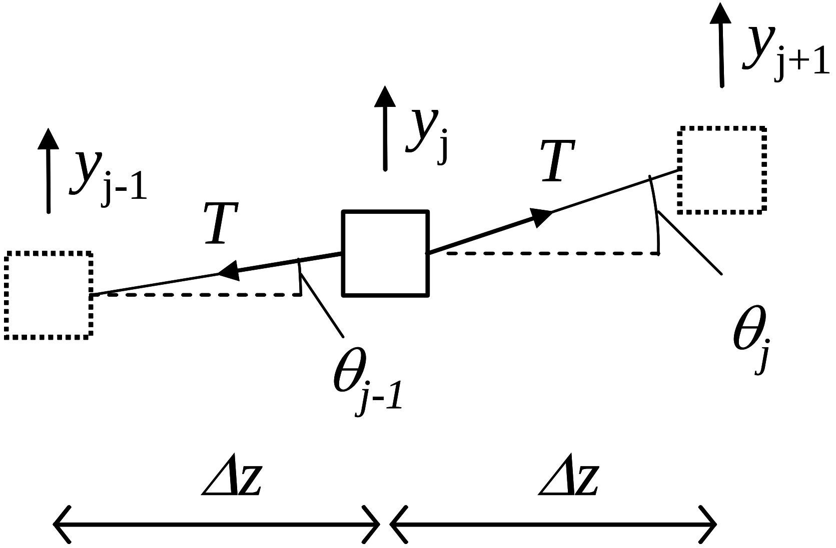
$$F = T\Delta z \frac{\partial^2 y}{\partial z^2}$$

we are saying that

the force F is proportional to the curvature of the "string" of masses

There is no net vertical force if the masses are in a straight line

Classical wave equation



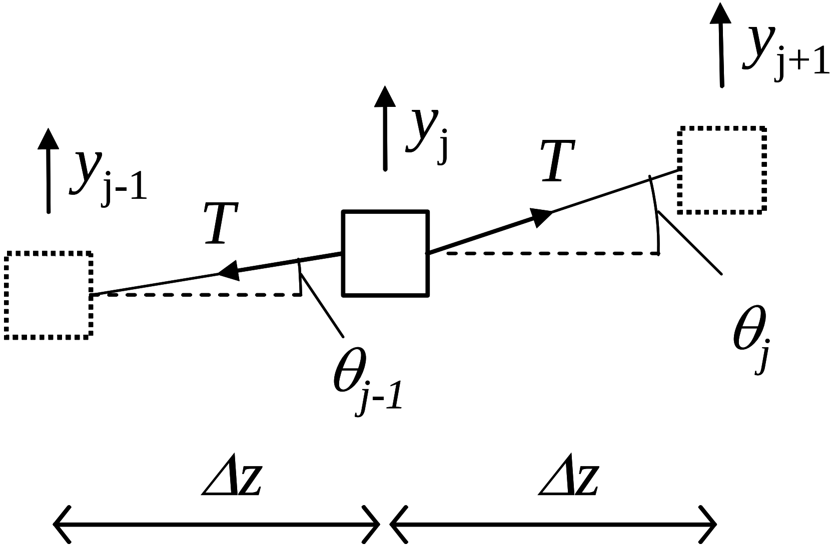
Think of the masses as

the amount of mass per unit length in the z direction, that is

the linear mass density ρ times Δz, i.e., $m = \rho \Delta z$

Then Newton's second law gives

$$F = m\frac{\partial^2 y}{\partial t^2} = \rho \Delta z \frac{\partial^2 y}{\partial t^2}$$



Classical wave equation

Putting together

$$F = T\Delta z \frac{\partial^2 y}{\partial z^2} \quad \text{and} \quad F = \rho\Delta z \frac{\partial^2 y}{\partial t^2}$$

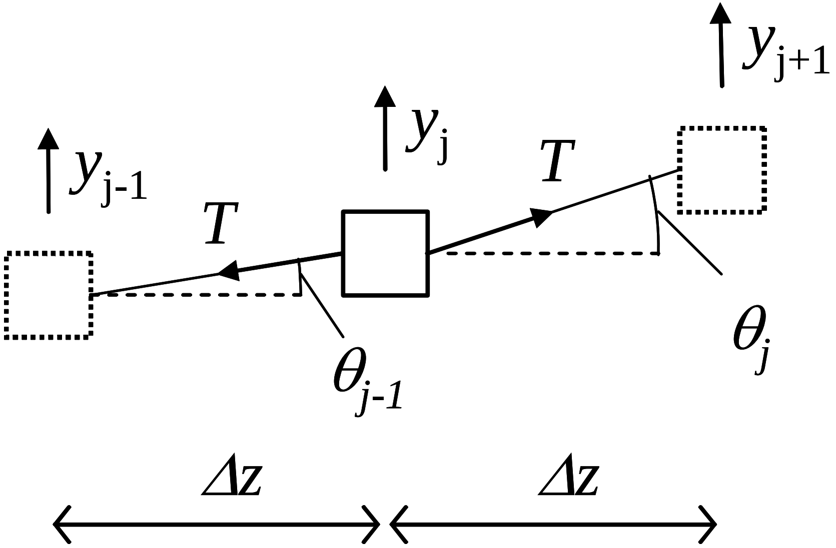
gives

$$T\Delta z \frac{\partial^2 y}{\partial z^2} = \rho\Delta z \frac{\partial^2 y}{\partial t^2}$$

i.e.,

$$\frac{\partial^2 y}{\partial z^2} = \frac{\rho}{T} \frac{\partial^2 y}{\partial t^2}$$

Classical wave equation



Rewriting

$$\frac{\partial^2 y}{\partial z^2} = \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2}$$

with

$$v^2 = T / \rho$$

gives

$$\frac{\partial^2 y}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

which is a wave equation for a wave with velocity $v = \sqrt{T / \rho}$

Wave equation solutions – forward waves

We remember that any function of the form $f\left(z-ct\right)$

is a solution of the wave equation

and is a wave moving to the right with velocity c

Wave equation solutions – backward waves

We remember that any function of the form $g(z+ct)$

is a solution of the wave equation

and is a wave moving to the left with velocity c

Monochromatic waves

Often we are interested in waves oscillating at one specific (angular) frequency ω

i.e., temporal behavior of the form

$$T(t) = \exp(i\omega t),\ \exp(-i\omega t),\ \cos(\omega t),\ \sin(\omega t)$$

or any combination of these

Then writing $\phi(z,t) \equiv Z(z)T(t)$, we have $\dfrac{\partial^2 \phi}{\partial t^2} = -\omega^2 \phi$

leaving a wave equation for the spatial part

$$\frac{d^2 Z(z)}{dz^2} + k^2 Z(z) = 0$$

where $k^2 = \dfrac{\omega^2}{c^2}$

the Helmholtz wave equation

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

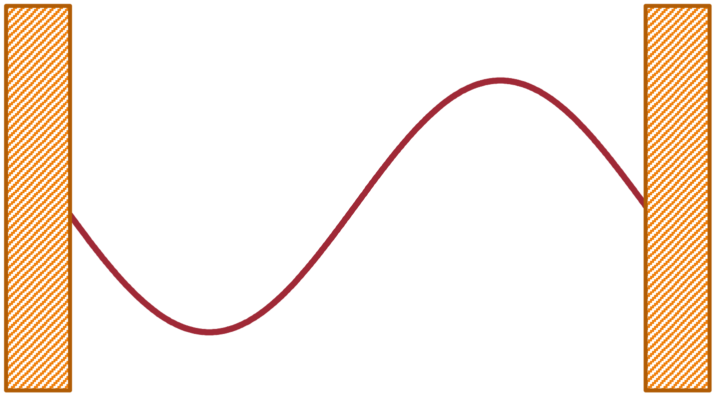
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

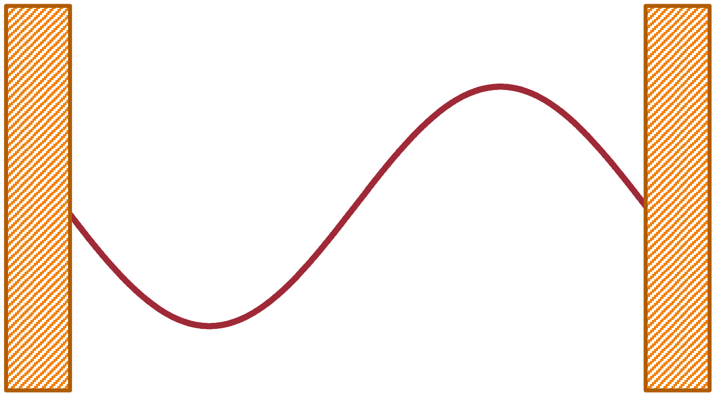
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

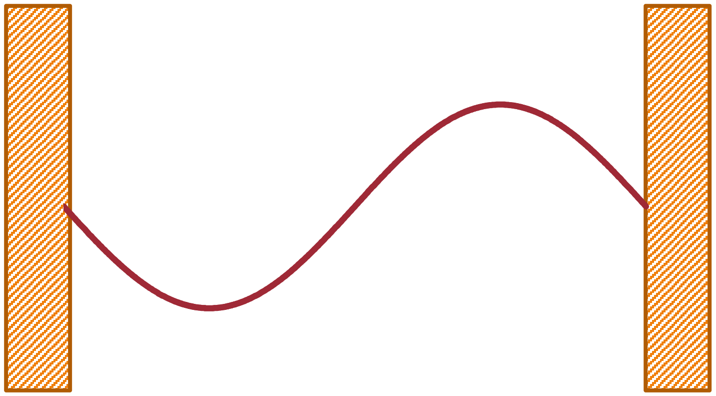
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

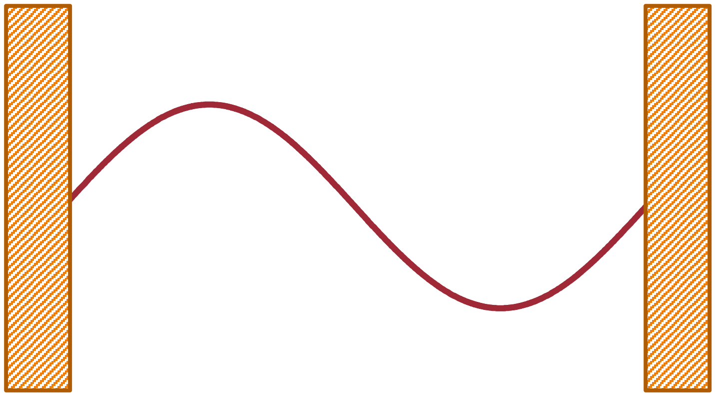
An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"



E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

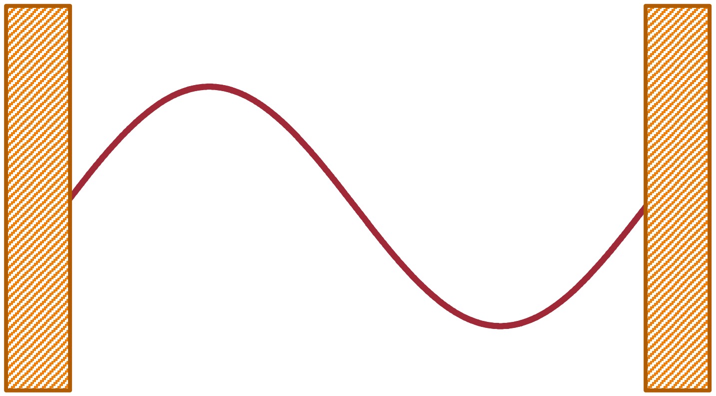
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

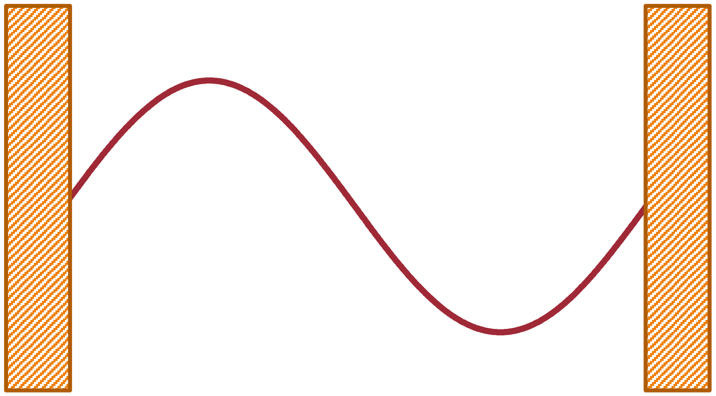
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

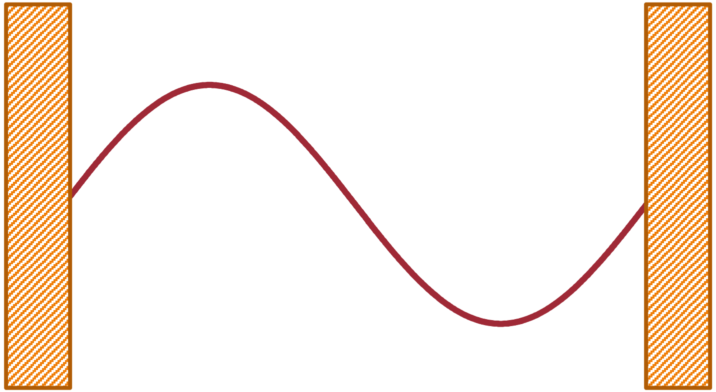
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

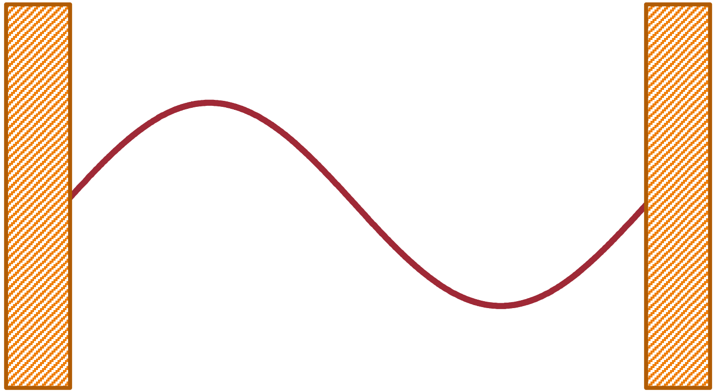
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$

Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

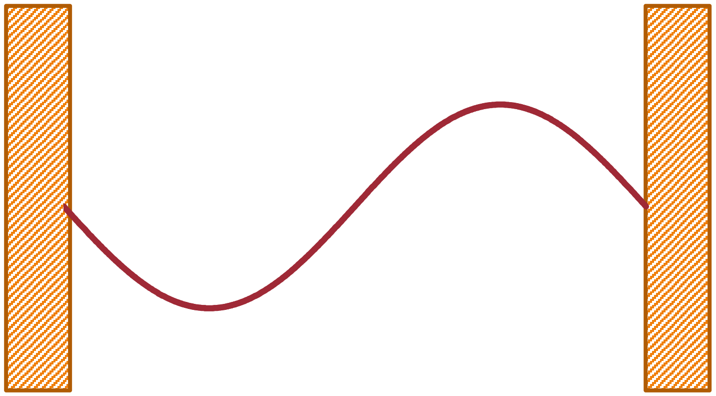
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

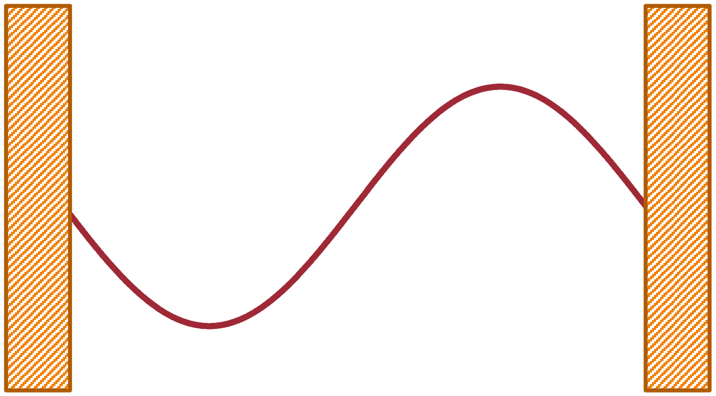
$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$



Standing waves

An equal combination of forward and backward waves, e.g.,

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$

$$\equiv 2\cos(\omega t)\sin(kz)$$

where $k = \omega / c$

gives "standing waves"

E.g., for a rope tied to two walls a distance L apart

with $k = 2\pi / L$ and $\omega = 2\pi c / L$