Measuring and processing partially coherent light with self-configuring optics

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Abstract: We show that self-configuring optical networks can analyze partially incoherent light. We consider the case of N spatial input channels and present a power-optimization method to measure their coherency matrix.

In optics and photonics, partially coherent light is the norm rather than the exception and accounts for emission processes in LEDs, thermal emitters, photovoltaics, luminescent and scintillating materials, as well as natural light for sensing the environment. The most general way to describe it is through its coherency matrix ρ [1,2], which can characterize such partial coherence over arbitrary channels of the system, e.g., spatial, temporal, spectral, or polarization degrees of freedom. Methods to reconstruct ρ for a few polarization-spatial channels have been demonstrated via projective measurements [3]. Despite the ubiquity of partial coherence in optical phenomena, there is no general, scalable method to measure ρ, nor to decompose it into its mutually incoherent parts.

Meshes of Mach-Zehnder interferometers (MZI) have proven very effective at manipulating [4] and measuring [5] coherent multi-mode light. Now, we propose that these self-configuring meshes can measure the full coherency matrix of partially coherent light, additionally separating it into its mutually incoherent orthogonal components.

Our method performs sequential power optimization over the N output channels of a self-configuring network, thereby learning the matrix eigenvectors, which can then be deduced directly from the resulting settings of the network elements. The corresponding eigenvalues can be measured by reading out the average values of the output intensities. In this process, the mesh network has also separated the incoming light into its mutually incoherent orthogonal components, and the unitary network is then also implementing the linear transform that diagonalizes the coherency matrix, effectively measuring this matrix. Our method therefore paves the way to full characterization and processing of partially coherent light.

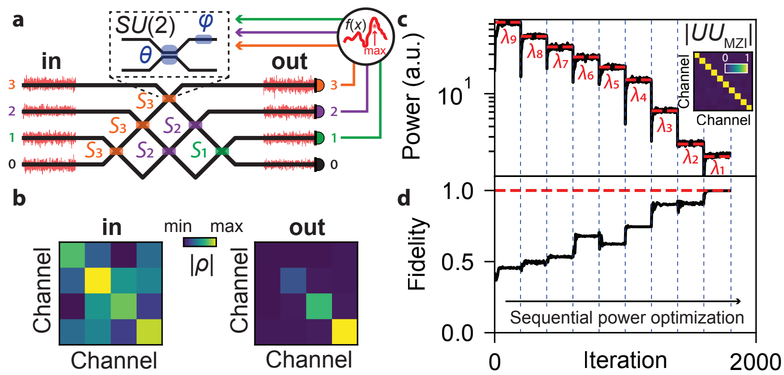

Fig. 1: **Processing partially coherent light with arrays of self-configuring Mach-Zehnder interferometers. a.** An incident wavefront of partially coherent waves (described by coherency matrix ρ) is sent through a triangular array of Mach-Zehnder interferometers (MZI). Each node of the array is a 2×2 MZI encoding a SU(2) matrix. A sequential power optimization diagonalizes the coherency matrix ρ, resulting in uncorrelated output signals. **b.** Example input and output coherency matrices, the latter resulting from the sequential power optimization. **c.** The power optimization routine sequentially maximizes the power at each output channel (with channel index i from 0 to $N-1$), mapping each output sequentially to the corresponding coherency matrix eigenvalue (λ_i). Inset: Matrix product showing that U_{MZI} learns $U^\dagger$ (up to a diagonal matrix of phases). **d.** Resulting fidelity over power optimization iteration.

We consider N "channels" of input light, whose amplitudes are denoted by a N-dimensional vector x. These channels can describe spatial, polarization, or even spectral modes of light x_i with partial coherence described by the coherency matrix ρ [1], such that $\rho_{ij} = \langle x_i x_j^* \rangle$, where $\langle\cdot\rangle$ denotes ensemble averaging. The matrix ρ is Hermitian and non-negative, so can be written as $\rho = UDU^\dagger$, where U is a unitary matrix, and D is a diagonal matrix of eigenvalues $\lambda_i \geq 0$. Characterizing ρ entails measuring the unitary operator U and the eigenvalues λ_i.

A linear operation U_{MZI} on these channels transforms the coherency matrix as : $\rho' = U_{\text{MZI}}\rho U_{\text{MZI}}^\dagger$ [2] (where $\rho' = \langle yy^\dagger \rangle$ and $y = U_{\text{MZI}}x$ is the network output). In the following, we consider linear transformations imparted by self-configuring networks of MZIs [6], such as the triangular network shown in Fig. 1a [7]. It is known that such networks can implement arbitrary linear transformations between inputs and outputs by automatically "learning" the corresponding singular value decomposition [6]. Each node of the network acts on two neighboring channels only and imparts a reconfigurable SU(2) matrix with parameters (θ, φ).

We devised a sequential power optimization algorithm that performs diagonalization of the coherency matrix ρ for a given set of inputs x_i. Each step of the algorithm consists in the maximization of the time-averaged power at one of the output ports (from $k = N-1$ to 1, with the convention of Fig. 1a). At step k, the network optimization is the following:

$$\max_{(\theta,\varphi)\in S_k} \langle y_k y_k^* \rangle = \lambda_k,$$

where λ_k is the k-th largest eigenvalue (ordered such that $\lambda_0 < \ldots < \lambda_{N-1}$), and S_k is the k-th diagonal of MZI nodes. For illustrative purposes, each node of the network in the 4-channel example of Fig. 1a is labelled with the corresponding output port optimization (with S_3 shown in orange, S_2 in purple, and S_1 in green, respectively). This algorithm is a direct consequence of the min-max or Courant-Fischer theorem for Hermitian matrices.

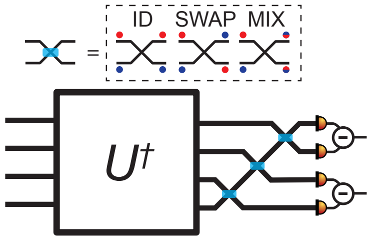

Fig. 2: **Analyzer circuit design.** The analyzer network consists in a single layer of nodes that can be configured in either one of three gates: identity, swap, and Hadamard gates (mixing). The output signals are then analyzed via balanced homodyne measurements.

One key for this algorithm is that the MZI network should consist of multiple self-configuring layers or rows [5, 6]. Then we can optimize the mesh settings sequentially for one row of MZIs at a a time, the relative power at output node k gives λ_k, and the unitary network "diagonalizes" the coherency matrix ρ (see Fig. 1b), such that $U_{\mathrm{MZI}} = U^\dagger$. So, reading out the network parameters and output powers fully characterizes the coherency matrix.

We demonstrate the validity of our approach with numerical experiments in Fig. 1c-d with a 10-channel partially incoherent input. The input fields x are parametrized as a 10-dimensional vector of random variables described by covariance matrix ρ. As the power optimization is carried out, each channel's output power gives the corresponding eigenvalue of ρ (in our example, the condition number of ρ is $\lambda_9/\lambda_0 = 1.3\times 10^3$). The corresponding fidelity $F = \langle |U_{\mathrm{MZI}}U|,\ \mathrm{Id}\rangle_{\mathrm{HS}}$ (where $\langle\cdot\rangle_{\mathrm{HS}}$ is the Hilbert-Schmidt dot product) increases throughout the power optimization and reaches values > 0.99. In our numerical implementation, gradients of the time-averaged output powers were calculated using automatic differentiation and optimized with stochastic gradient descent [8]. In experimental implementations, various gradient calculation or measurement techniques could be used, such as *in situ* back-propagation [9] or dithering [4].

Once configured, the fields in different output channels of this network should be mutually incoherent; if we then attempt to interfere each pair of outputs, we should see no interference between them as the relative phase of those outputs is varied. To test such mutual incoherence, one can use an additional output analyzer layer of MZIs, as in Fig. 2, after the coherency diagonalization circuit $U_{\mathrm{MZI}} = U^\dagger$. To interfere any two outputs, the MZI nodes can be appropriately configured as (1) identity; (2) swap; (3) or mix (i.e., 50:50 splitter, as in Hadamard gates), shown in Fig. 2, onto an output photodetector. Scanning the relative input phase using the analyzer input phase shifters should then produce no interference fringes, which is equivalent to performing balanced homodyne measurements, yielding a zero-mean power $\langle y_i y_j^\dagger \rangle = 0$.

In conclusion, we have shown that self-configuring photonic networks, such as triangular arrays of MZIs, can automatically learn and measure the coherency matrix of an input wave across N channels. We envision that this method could process and analyze the coherency of light and matter in partially coherent light emission processes and sensing.

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