Optics à la mode - a new way of making, using and understanding optics

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Optics à la mode?

Optics à la mode

- served with ice cream?
  - No!
- in the most modern style or fashion?
  - Sort of …
    - Noting too that fashions change
      - so our "optics à la mode" would have to be able to change to suit the "fashion"
        - programmable optics
        - and even self-configuring optics!
- a pun on the word "mode"
  - Yes!

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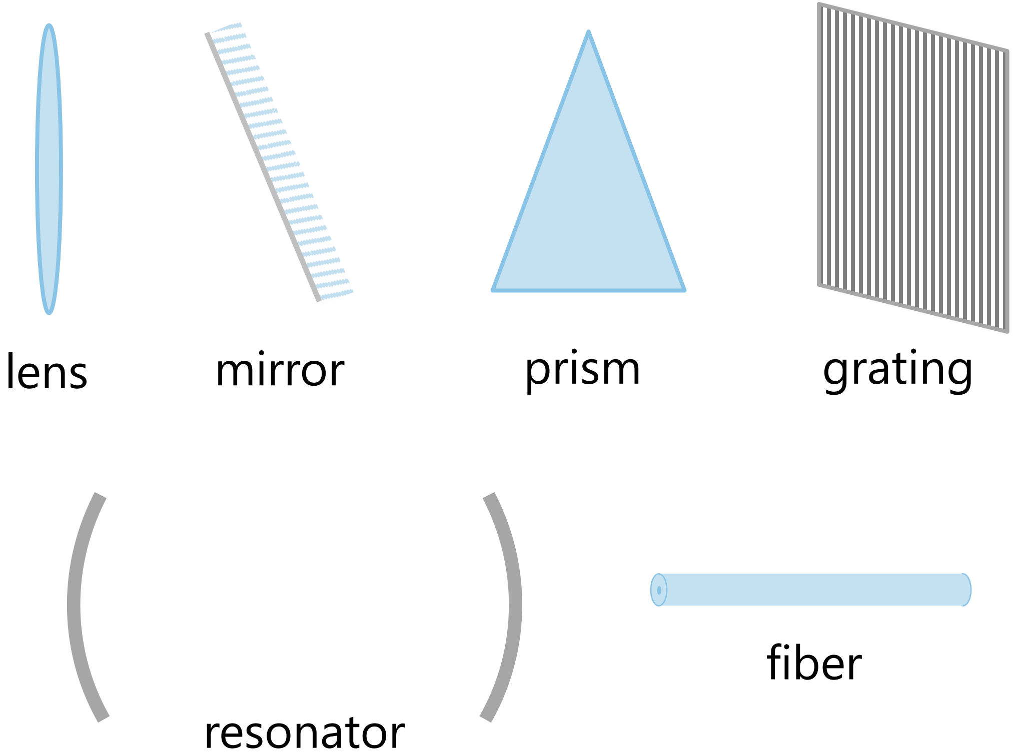
Optical components

Classic optics

- rays, imaging, plane waves
- basic spectroscopy

Lasers and optical fiber communications

- add “modes”



New optical circuits

This silicon photonics circuit

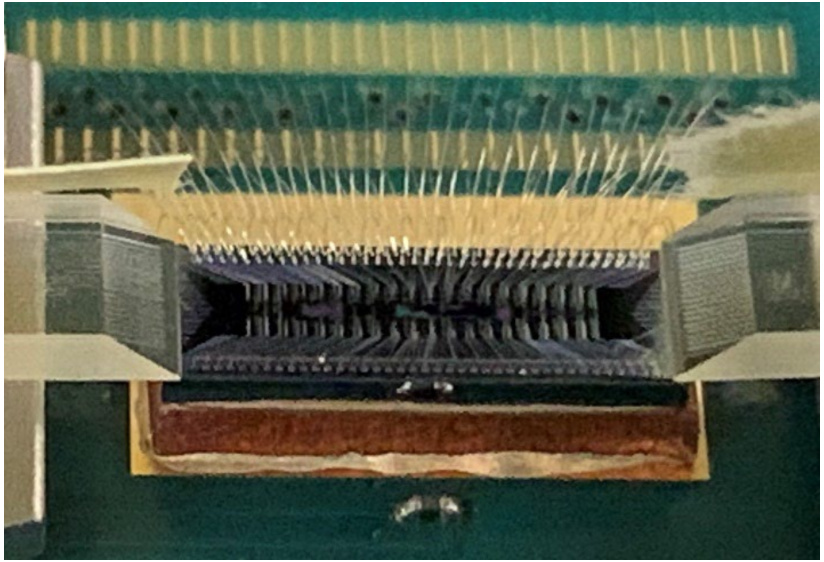
is also a piece of optics

- with an array of fibers as its input
- and an array of fibers as its output
- and the ability to arbitrarily define the relation between input and output

How do we think about this as optics?

Is there a broader approach that includes this as well as existing optics

and gives us new insights and tools?



Sunil Pai et al., Stanford

Universal matrix multiplier chip

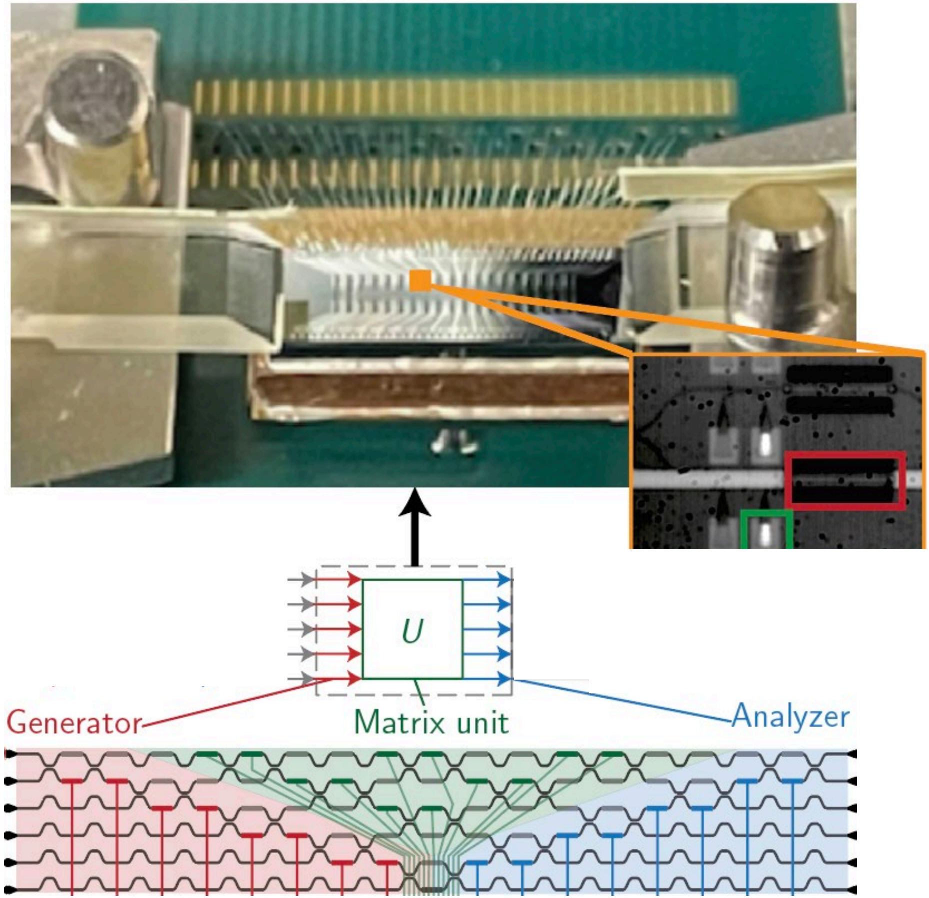
Universal matrix multiplying chip

"4x4" unitary Mach-Zehnder mesh with

- a "generator" to create any complex input vector
- an "analyzer" to measure the complex output vector

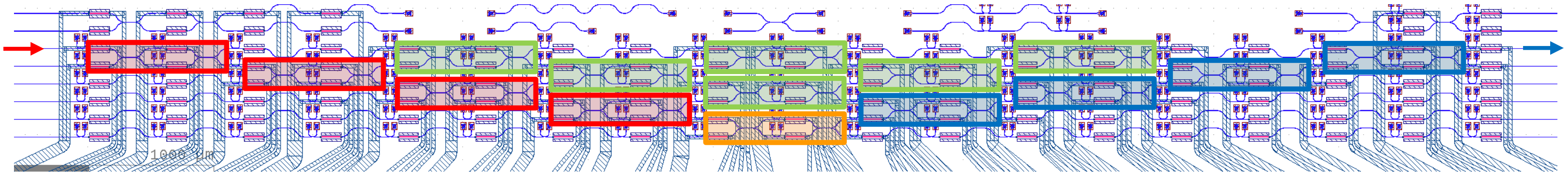
This can be programmed to implement

any "unitary" (loss-less) transformation from the inputs to the outputs



Mask layout and block diagram

Matrix unit



Vector generator

Vector analyzer

Universal matrix multiplier chip

Full complex matrix multiplication

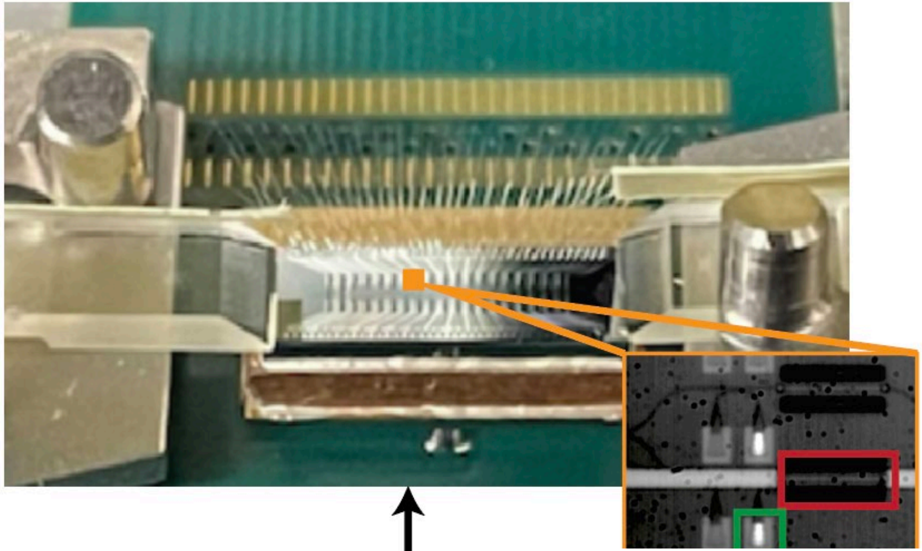
with vector generation and vector analysis

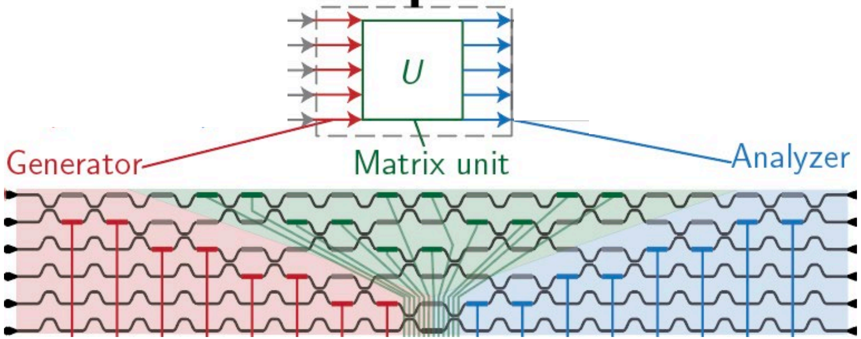
Photonic back-propagation neural net training

S. Pai, Z. Sun, T. W. Hughes, T. Park, B. Bartlett, I. A. D. Williamson, M. Minkov, M. Milanizadeh, N. Abebe, F. Morichetti, A Melloni, S. Fan, O. Solgaard, D. A. B. Miller, "Experimentally realized in situ backpropagation for deep learning in photonic neural networks," **Science** 380, 398-404 (2023)

Digital matrix multiplication for cryptography

S. Pai, T. Park, M. Ball, B. Penkovsky, M. Dubrovsky, N. Abebe, M. Milanizadeh, F. Morichetti, A. Melloni, S. Fan, O. Solgaard, and D. A. B. Miller, "Experimental evaluation of digitally verifiable photonic computing for blockchain and cryptocurrency," **Optica** 10, 552-560 (2023)





Self-configuring beam separator

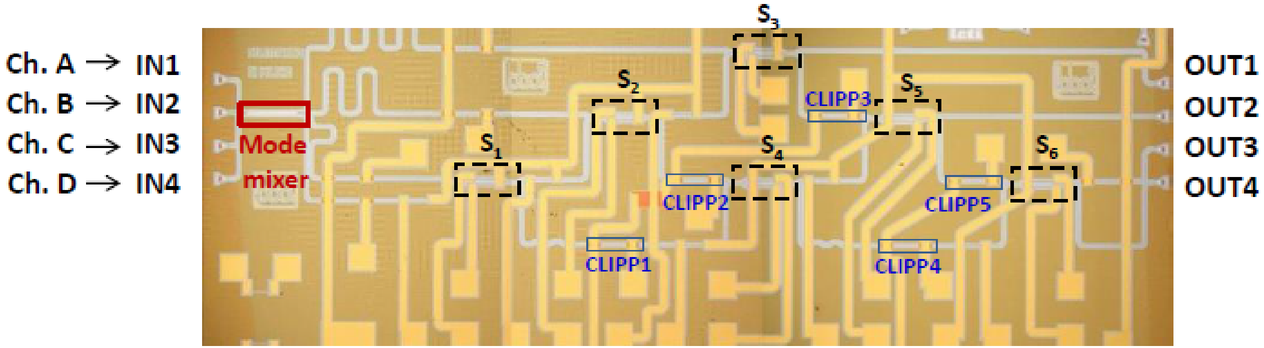
Light from four input fibers

deliberately mixed in a mode mixer

are automatically separated out again by a mesh of interferometers

by sequential power maximizations

without calculations



A. Annoni et al., "Unscrambling light – automatically undoing strong mixing between modes," Light Science & Applications 6, e17110 (2017)

See, e.g., review W. Bogaerts et al., "Programmable photonic circuits," Nature 586, 207 (2020)

Truly arbitrary linear optics

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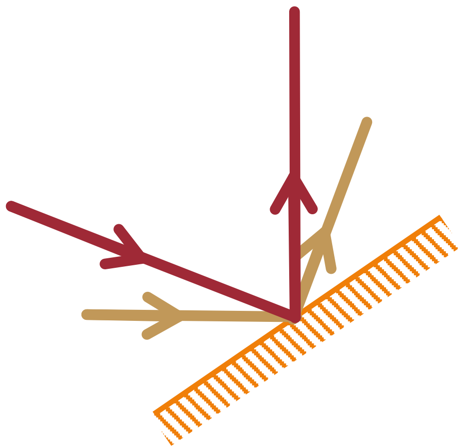
Simple optical components – a mirror

- We "design" a plane mirror
  - by choosing its angle
    - so it takes a beam of one angle
      - and changes it into a beam of another angle
- For another beam at another angle
  - the mirror changes it to a beam of yet another angle
    - but we have no independent control
      - over what happens for the second beam



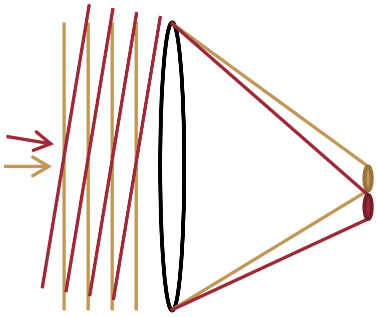
Simple optical components – a lens

We design a lens

- by choosing its index and curvatures
  - so it takes a plane wave in one direction
    - and focuses it to a spot

For another plane wave in another direction

- the lens focuses it to another spot
  - but we have no independent control
    - over what happens for the second beam



Simple "thin" optical components

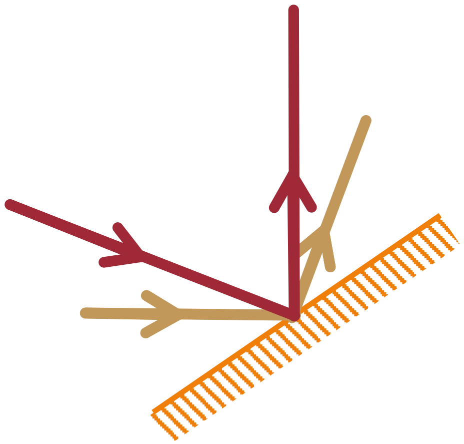
This kind of behavior is general for "thin" optical components

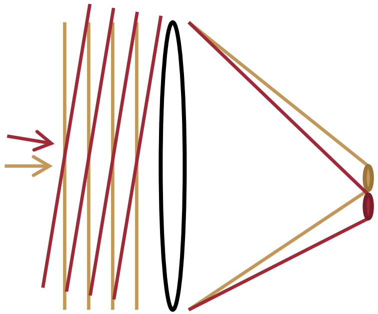
- e.g., thin holograms, diffractive optical elements
- spatial light modulators, adaptive optics,
- metasurfaces

We design them to perform some useful function for one input beam

- but we have no independent control
  - of what happens for other beams

So these are not "arbitrary" optical components





Example - Separating overlapping beams

Suppose we have two different (orthogonal) beams

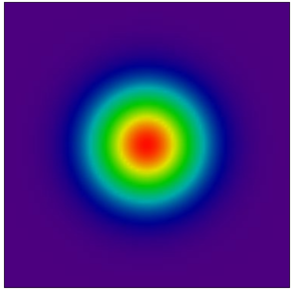
e.g., from an optical fiber

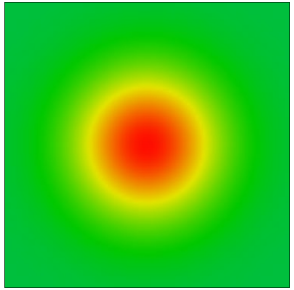
such as

a “single bump” beam

intensity

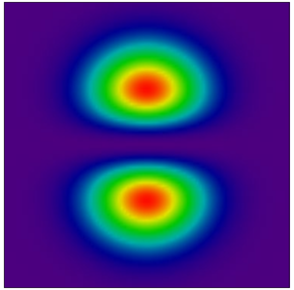
field

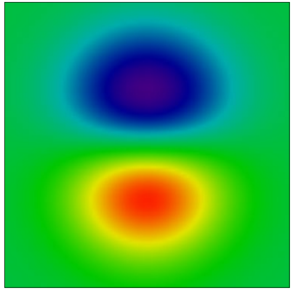




and

a “two bump” beam





Example - Separating overlapping beams

Mathematically,

two (non-zero) beams are "orthogonal" if

$$\iint \mathbf{E}_1^*(x,y)\cdot\mathbf{E}_2(x,y)\,dx\,dy = 0$$

Here, the product of the

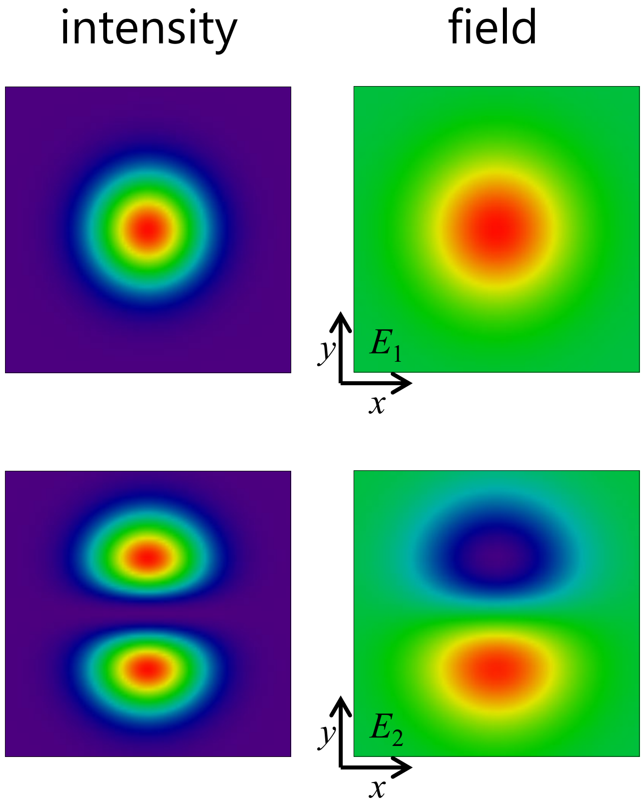
single-bump beam and the

two-bump beam

would be negative in the top half

but positive in the bottom half

so the resulting integral would be zero



Example - Separating overlapping beams

If both of these beams emerge simultaneously from the fiber

how can we separate them

for example to different fibers

without loss?

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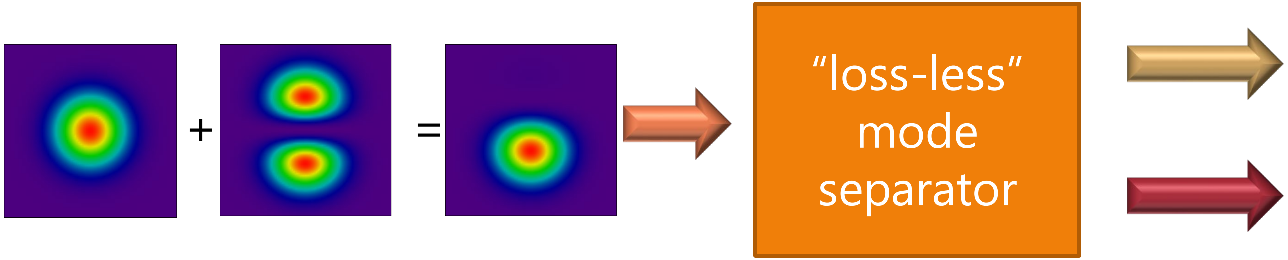


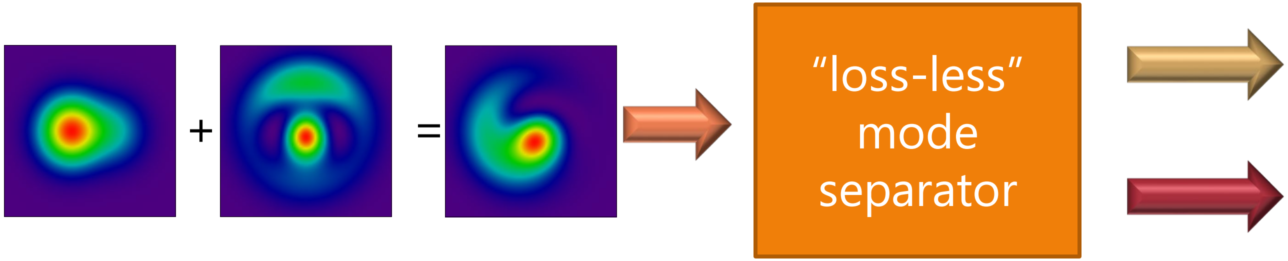
Example - Separating overlapping beams

In situations with

fixed

highly symmetric beams

good specific low-loss separation solutions are known



But for general cases

of lower symmetry and/or higher complexity

or where the beams change in time

general solutions have not been known

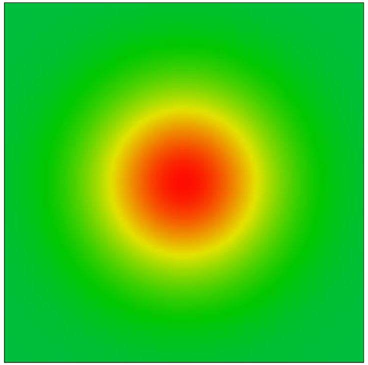
Dividing the beam into "patches"

We can approach the beam-separation problem

by presuming it will be good enough

to imagine that we can divide the beam

into a finite number of "patches"



Dividing the beam into “patches”

We can approach the beam-separation problem

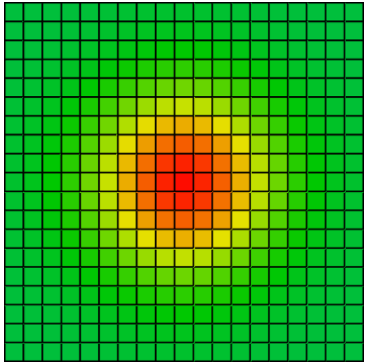
- by presuming it will be good enough
  - to imagine that we can divide the beam
    - into a finite number of “patches”

We treat each of these patches

- as if it was approximately uniform
  - in intensity and
    - in phase

At least with a sufficiently large number of patches

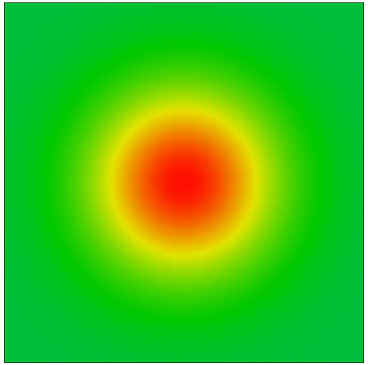
- this could be a good enough approximation
  - and “sampling loss” may be small

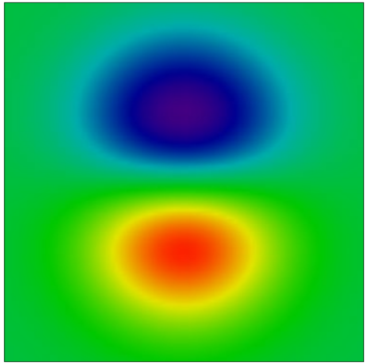


Dividing the beam into "patches"

Even relatively small numbers of patches

- are sufficient to distinguish beams of low or moderate complexity





Dividing the beam into "patches"

Even relatively small numbers of patches

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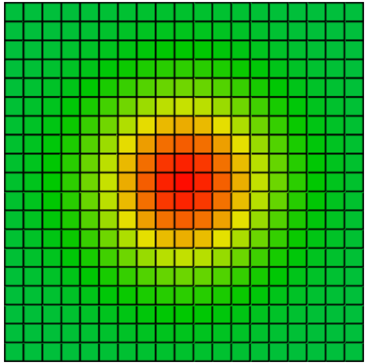

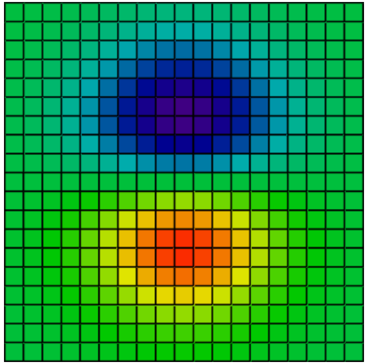

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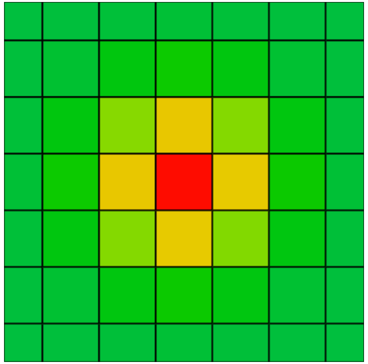

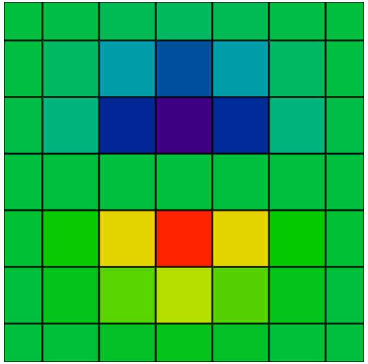

Separating beams

So, by dividing the beam into patches

- we may be able to approximate the problem
  - by one of finite dimensions

Indeed, the approach discussed here

- will only be practical for problems of limited dimensionality
  - e.g., 10's or possibly 100's

But still, even in principle

- how can we separate these beams?

Mach-Zehnder interferometer meshes

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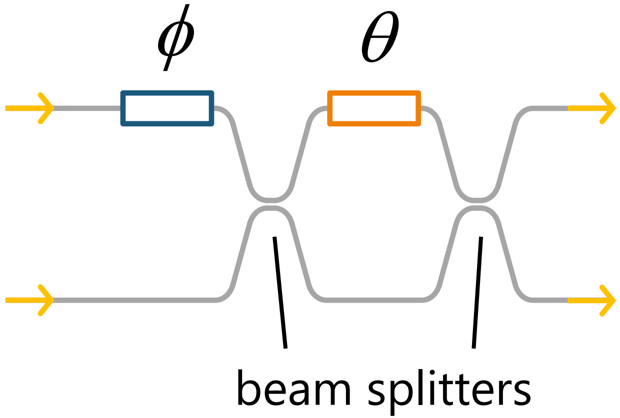
Nulling a Mach-Zehnder output

Consider a waveguide Mach-Zehnder interferometer (MZI)

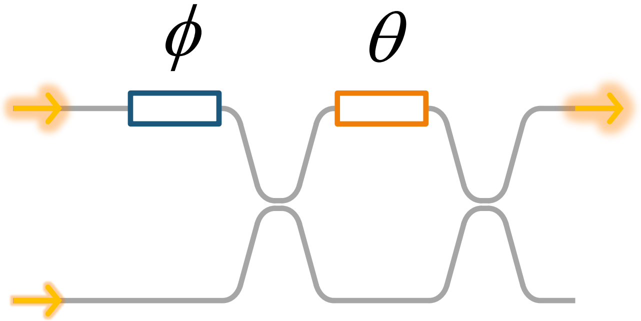
- formed from two "50:50" beam splitters
- and at least two phase shifters
  - one, ϕ, to control the relative phase of the two inputs
  - a second, θ, to control the relative phase on the interferometer "arms"



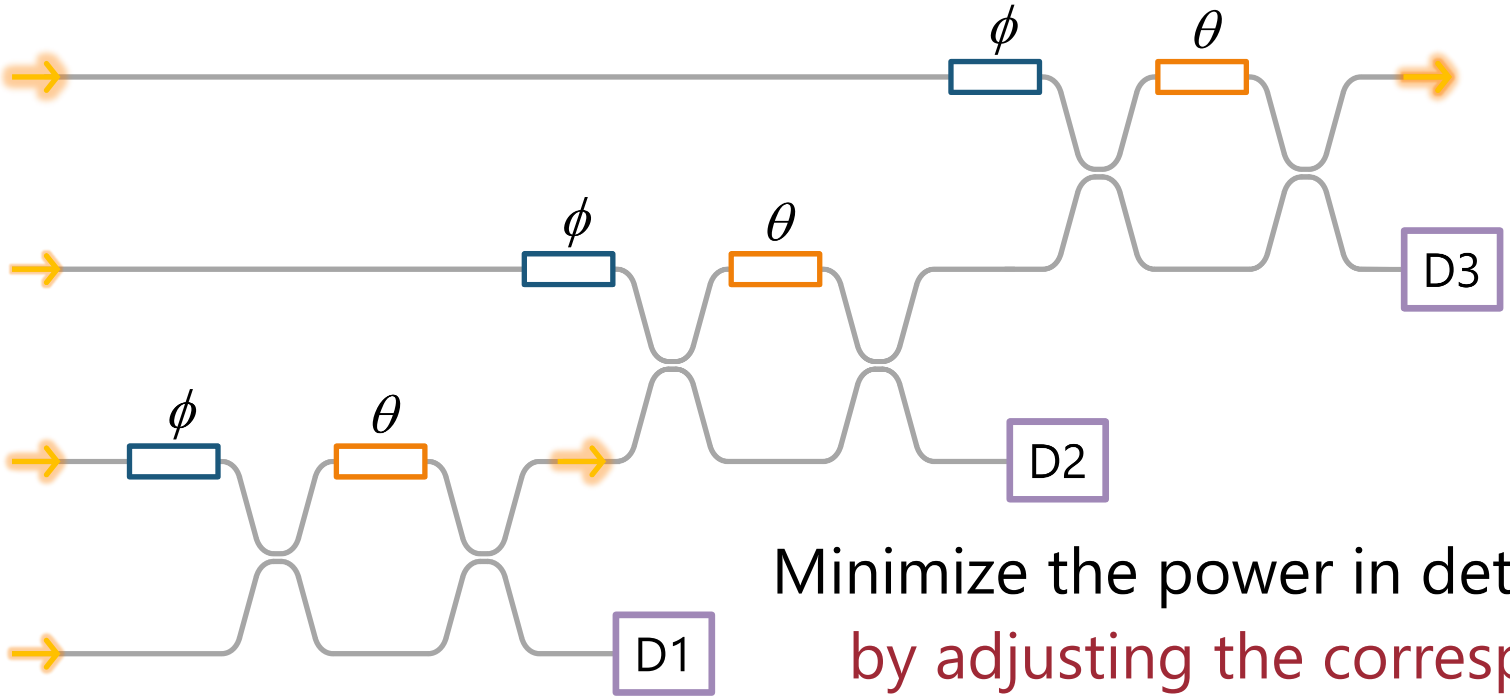
Nulling a Mach-Zehnder output

In such an MZI with 50:50 beamsplitters

- for any relative input amplitudes and phases
  - we can "null" out the power at the bottom output
    - by two successive single-parameter power minimizations
      - first, using ϕ
      - second, using θ



"Diagonal line" self-aligning coupler



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

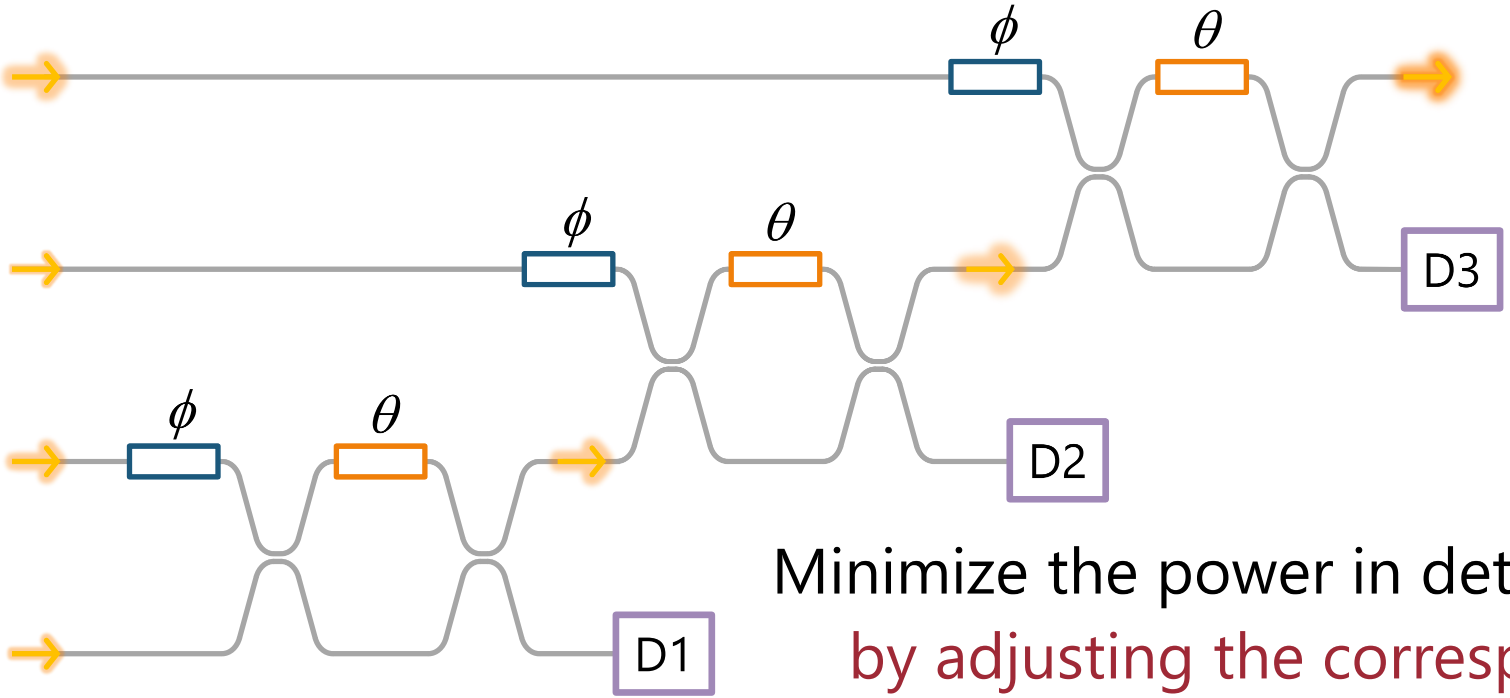
Minimize the power in detector D1

by adjusting the corresponding ϕ

and then θ

putting all power in the upper output

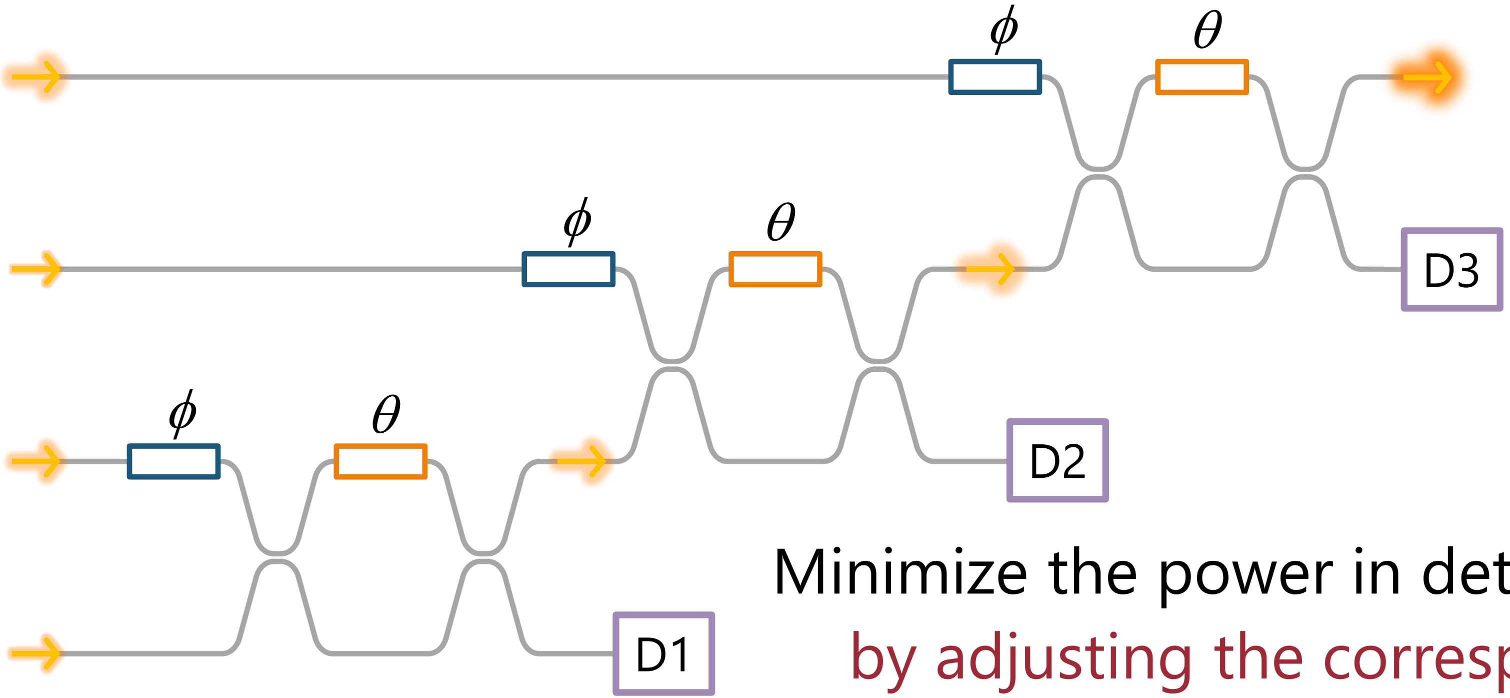
"Diagonal line" self-aligning coupler



Minimize the power in detector D2
by adjusting the corresponding ϕ
and then θ
putting all power in the upper output

"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

"Diagonal line" self-aligning coupler



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

Minimize the power in detector D3

by adjusting the corresponding ϕ

and then θ

putting all power in the upper output

Self-aligning beam coupler

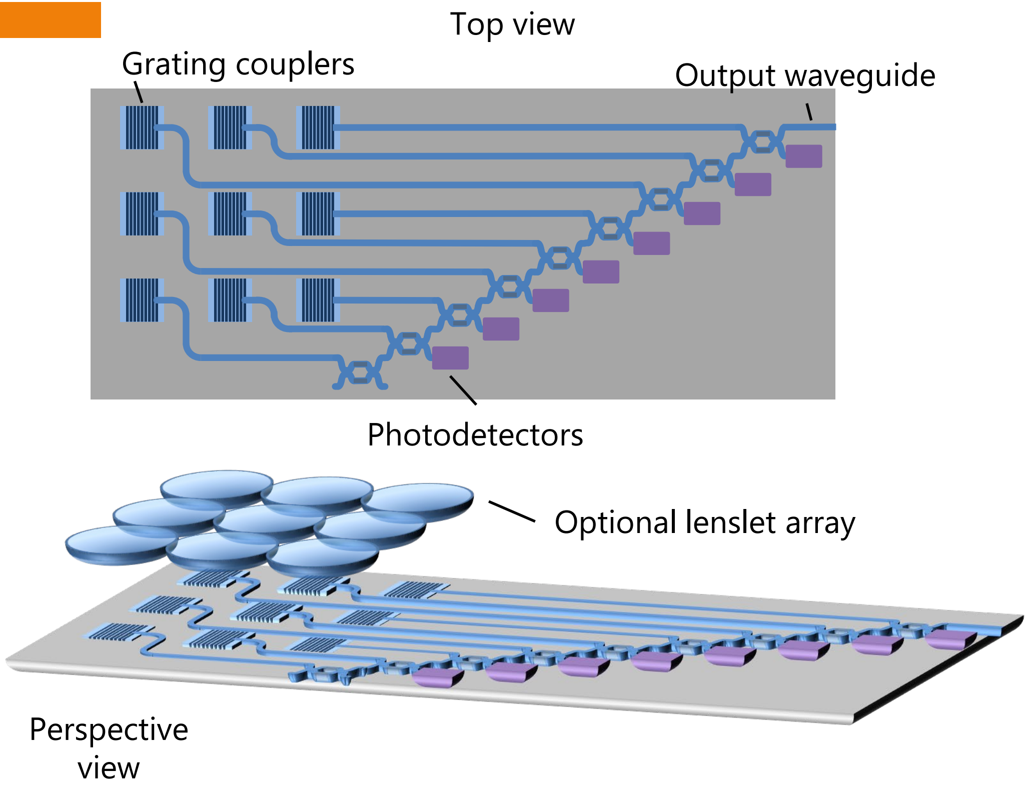
Grating couplers could couple a free-space beam to a set of waveguides

Then

we could automatically couple all the power to the one output guide

This could run continuously

tracking changes in the beam

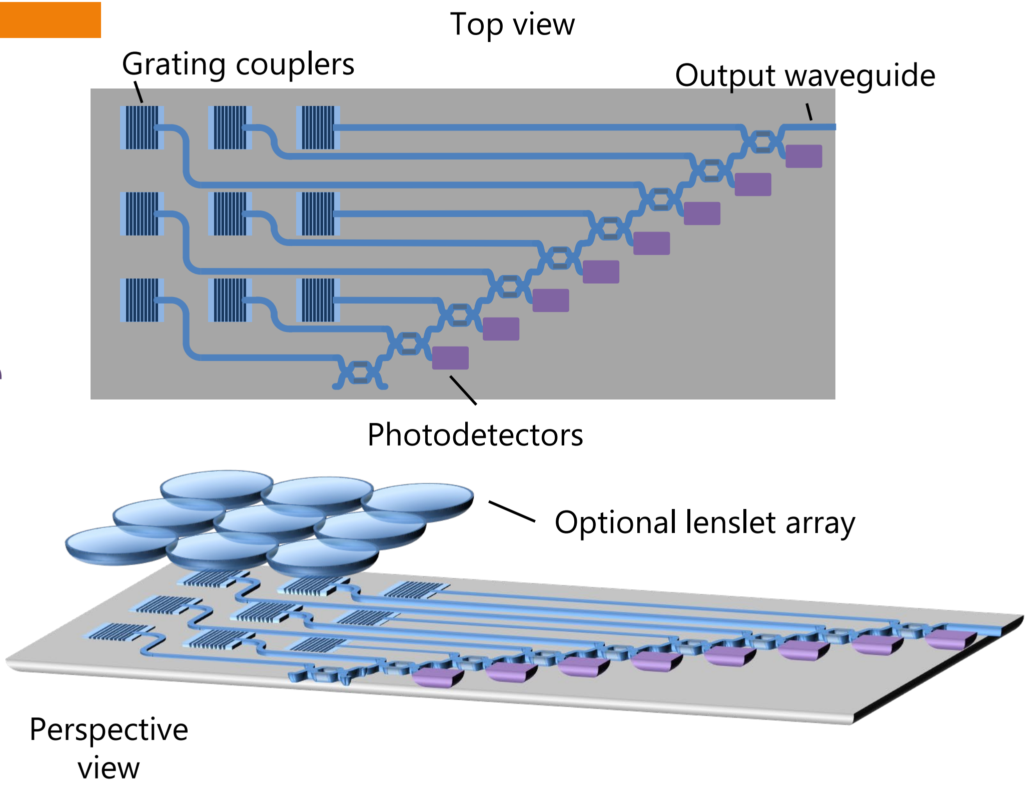


"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

Self-aligning beam coupler

This has several different uses

- tracking an input source
  both in angle and focusing
- correcting for aberrations
- analyzing amplitude and phase of the components of a beam
  - …



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

Separating beams with interferometer meshes

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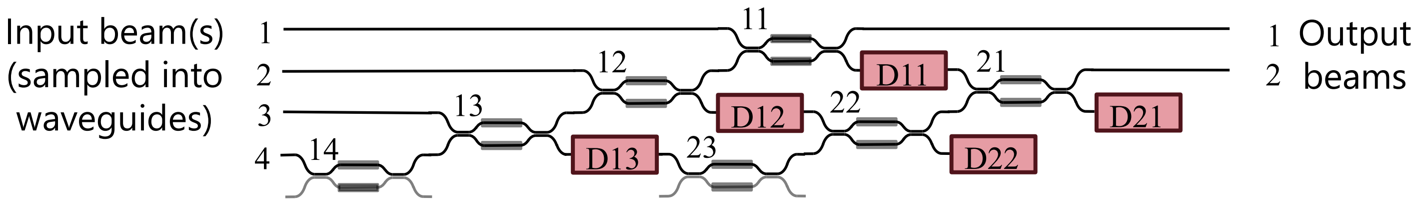
Separating multiple orthogonal beams



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

Once we have aligned beam 1 to output 1 using detectors D11 – D13

an orthogonal input beam 2 would pass entirely into the detectors D11 – D13

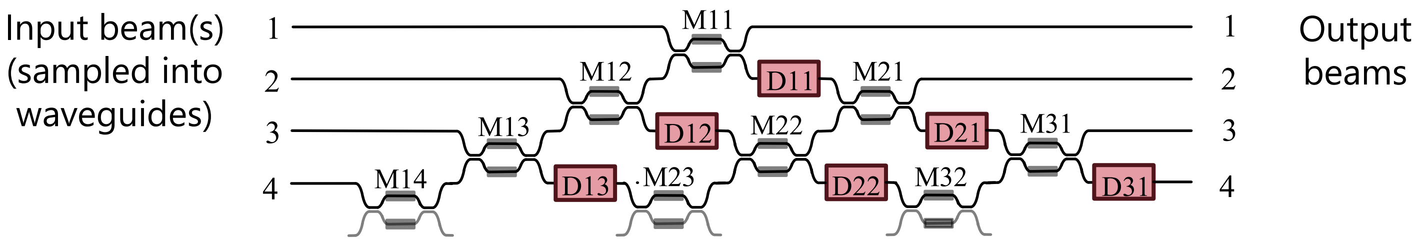
If we make these detectors mostly transparent

this second beam would pass into the second diagonal "row"

where we self-align it to output 2 using detectors D21 – D22

separating two overlapping orthogonal beams to separate outputs

Separating multiple orthogonal beams



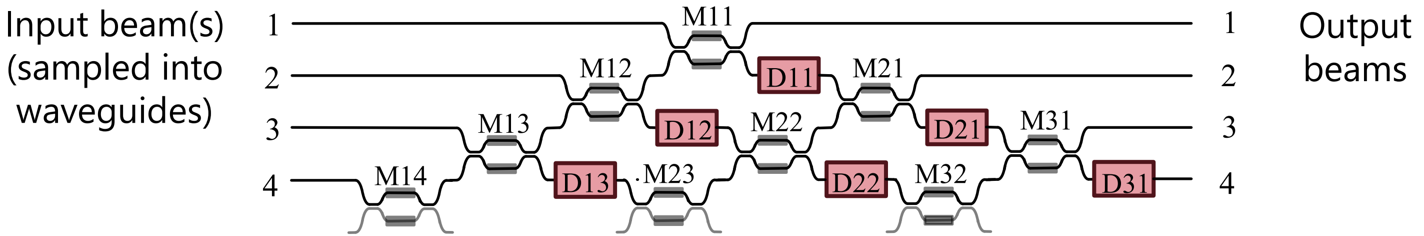
"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

Adding more rows and self-alignments

separates a number of orthogonal beams

equal to the number of beam "segments", here, 4

Separating multiple orthogonal beams



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

If we put identifying "tones" on each orthogonal input "beam"

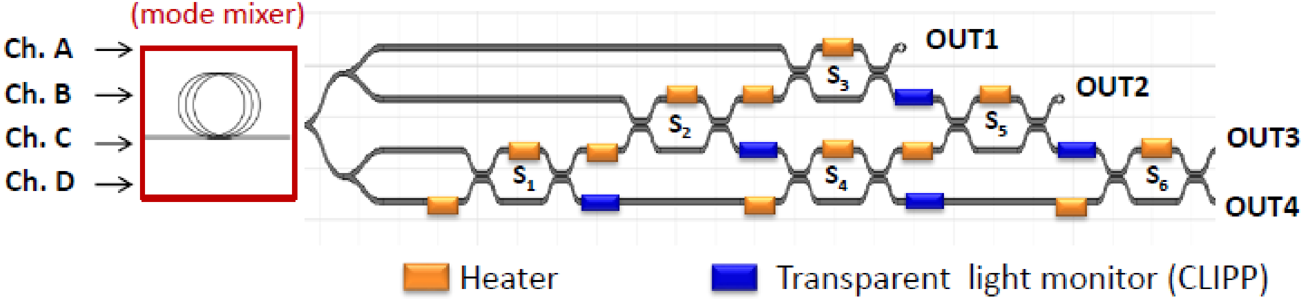
and have the corresponding diagonal row of detectors look for that tone

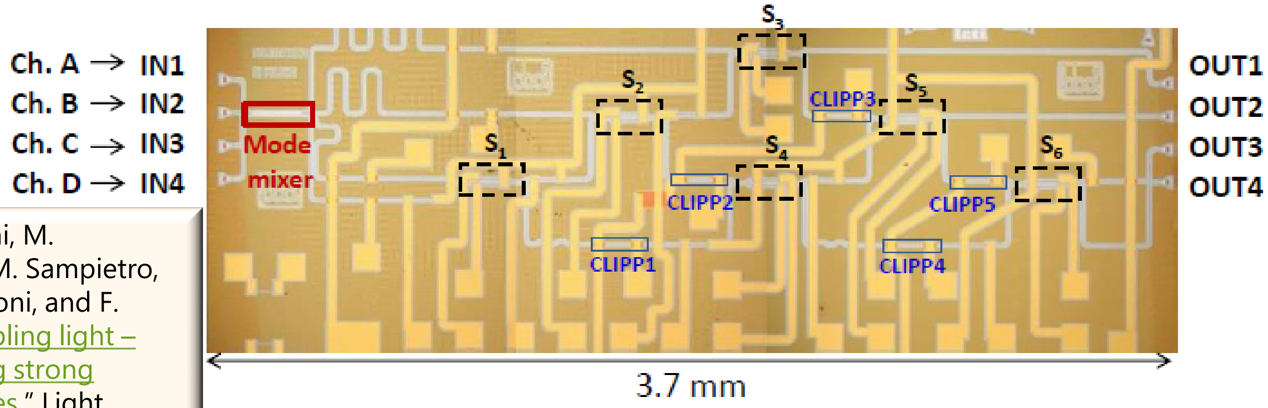
then the mesh can continually adapt to the orthogonal inputs

even when they are all present at the same time

and even if they change

Integrated MIMO demultiplexer: technology





A. Annoni, E. Guglielmi, M. Carminati, G. Ferrari, M. Sampietro, D. A. B. Miller, A. Melloni, and F. Morichetti, "Unscrambling light – automatically undoing strong mixing between modes," Light Science & Applications 6, e17110 (2017)

- Transparent detectors required for sequential tuning
- CLIPP-assisted circuit reconfiguration & feedback control

Si chip fabricated by CEA-Leti

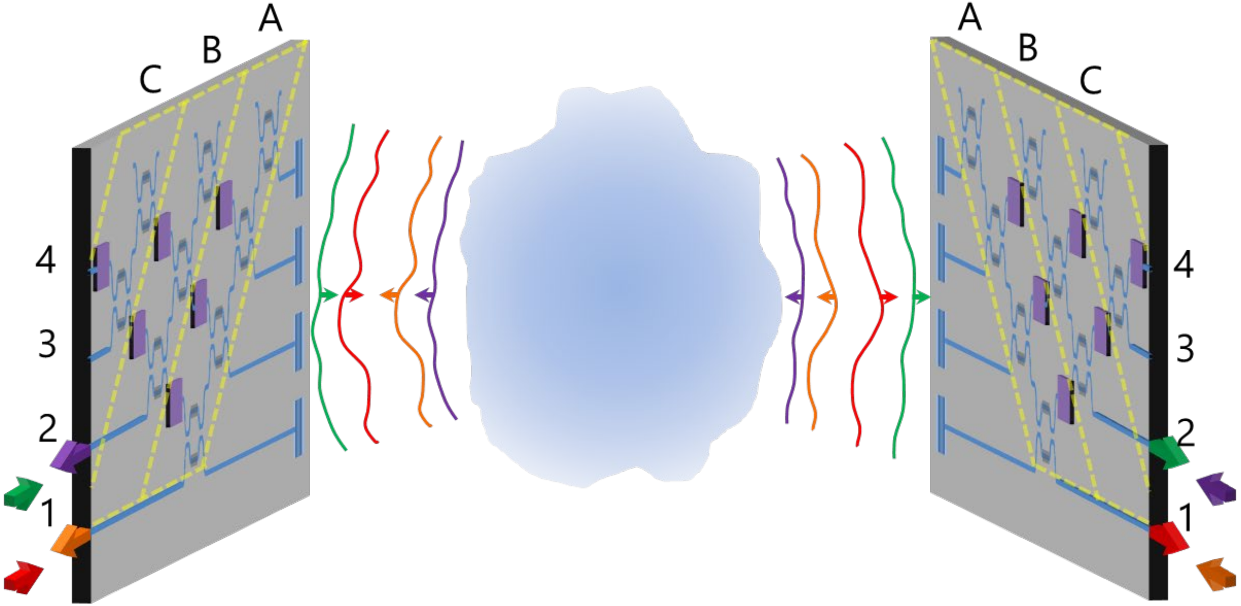
Establishing optimum orthogonal channels

In this architecture, using meshes on both sides

we proposed we could find optimal orthogonal channels through a scatterer

between waveguides on the left and waveguides on the right

by iterating back and forward between the two sides



"Establishing optimal wave communication channels automatically," J. Lightwave Technol. 31, 3987 (2013)

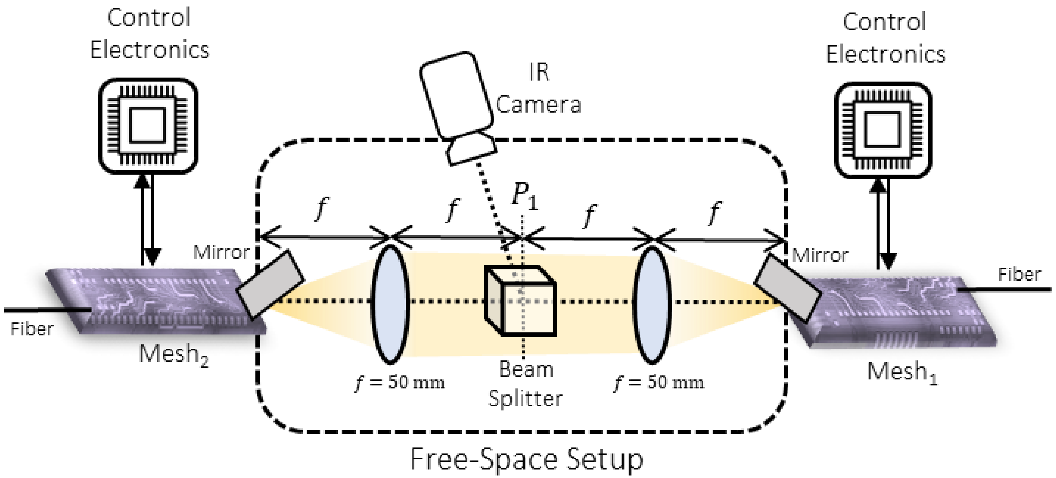
Establishing optimum orthogonal channels - experiment

We have now demonstrated this with two "facing" interferometer meshes with
- arbitrary optics between them

The optics can be misaligned
- and we can introduce
  - aberrations or partial blocking
- in the path

The system still self-aligns to find
- the best, orthogonal channels

This uses simple power optimizations
- on one "channel" at a time
  - at the output of each line of interferometers at the receiving end



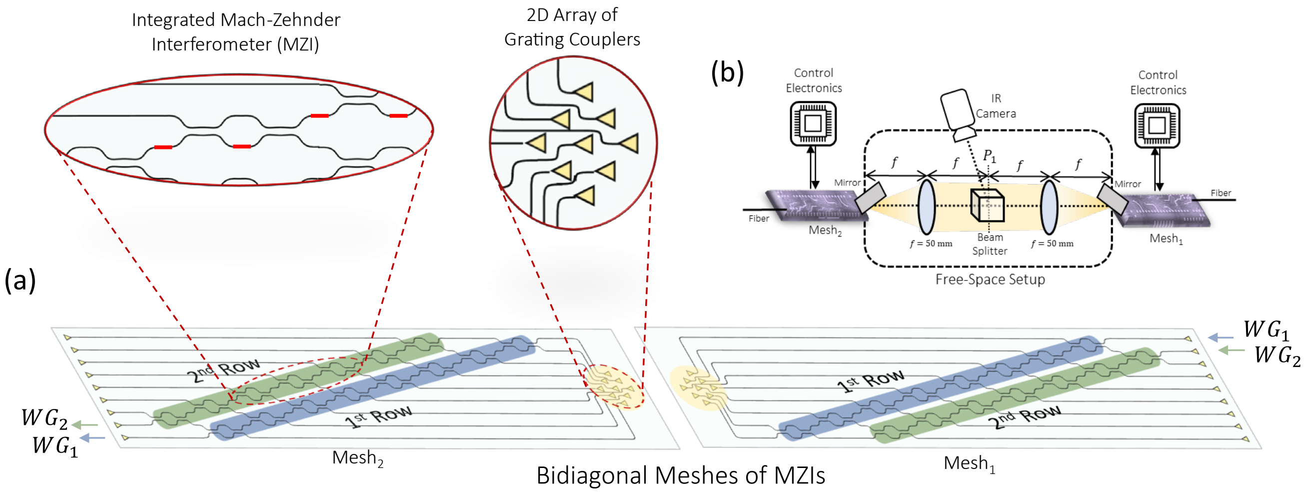
S. SeyedinNavadeh, M. Milanizadeh, F. Zanetto, G. Ferrari, M. Sampietro, M. Sorel, D. A. B. Miller, A. Melloni, and F. Morichetti, "Determining the optimal communication channels of arbitrary optical systems using integrated photonic processors," Nat. Photon. **18**, 149-155 (2024)

Establishing optimum orthogonal channels - experiment

Two “9x2” meshes allow automatic self-configuration

signals in WG1 on the right can automatically be aligned to appear out of WG1 on the left, and, at the same time

signals in WG2 on the right can automatically be aligned to appear out of WG2 on the left



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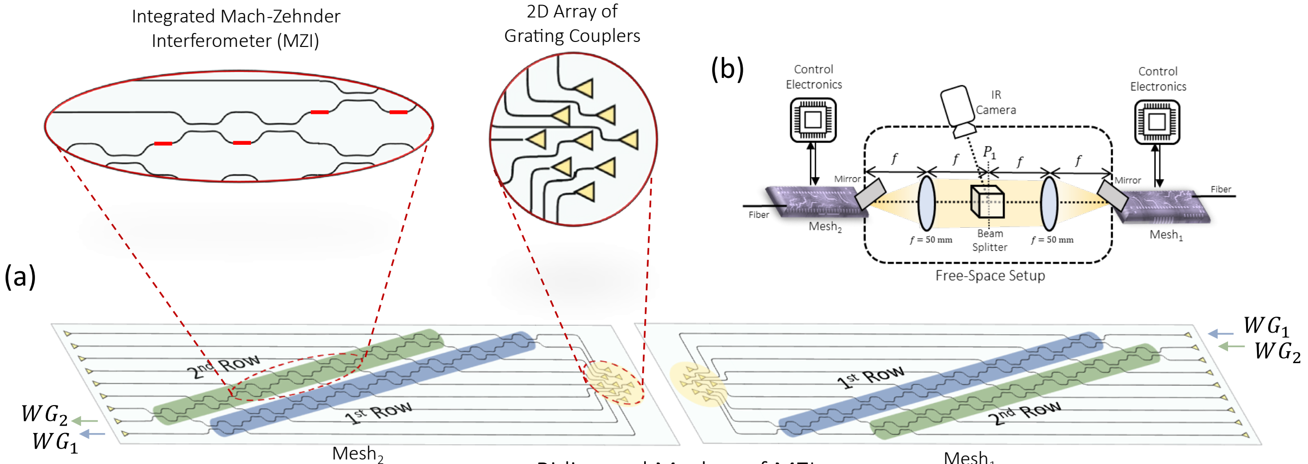
Establishing optimum orthogonal channels - experiment

Even after inserting a partially blocking mask in the optical path between the meshes

the system can re-establish orthogonal channels automatically

with > 30 dB rejection between the channels



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A new way of looking at optics

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A new way of looking at optics

This new way

- reproduces existing results
  - such as the limits from diffraction
- resolves paradoxes about the number of channels for communication
- clearly defines such channels
- gives us new physical laws
  - that only emerge from this view
    - e.g., new “Kirchhoff” radiation laws
    - new “modal” Einstein “A&B” law

Modal optics

Ray optics

Imaging optics

Fourier optics

Resonator modes

Waveguide modes

Beams

Modal optics

Full electro-magnetic simulation

We want a "modal" optics

to give the "right" way to describe optical systems

The optimal sets of functions

which even have basic physical laws that apply only to them

that give the most economical way to describe systems

including the "right" number of the "right" functions

To do this properly

we need to move beyond "resonator" and "waveguide" modes

and even beyond standard "beams"

"Waves, modes, communications, and optics: a tutorial," Adv. Opt. Photon. **11**, 679-825 (2019)

A different way of thinking about modes and waves

We are used to modes for

  resonators

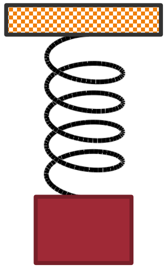
  propagating modes in waveguides

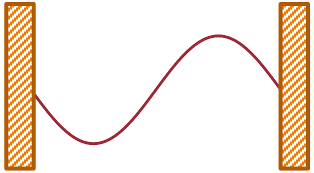
We like "modes" because they are economical

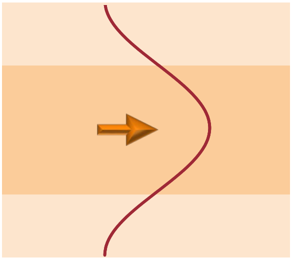
  We can use a few mode amplitudes

    not fields at every point

We can often "count" modes meaningfully







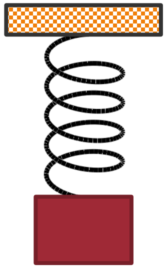
A different way of thinking about modes and waves

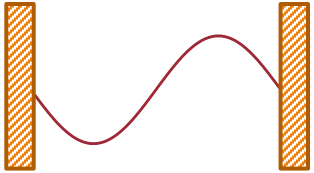
Modes have very useful mathematical properties, e.g.,

orthogonality

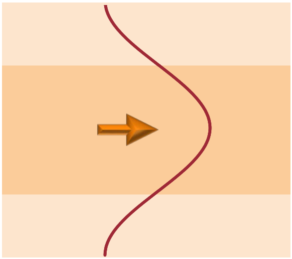
completeness

We can give a definition of a mode





A mode is an eigenfunction of an eigen problem describing a physical system



A different way of thinking about modes and waves

But when we look generally at

communications with waves

or scatterers, optical devices, or nanostructures

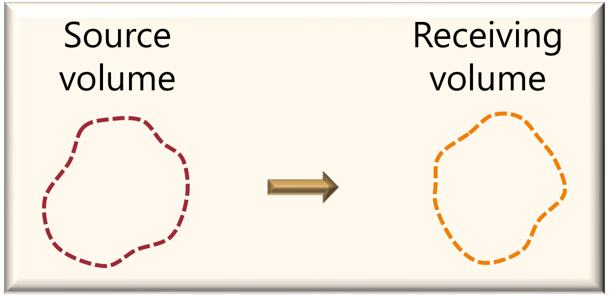
we need a different kind of "mode" that looks at

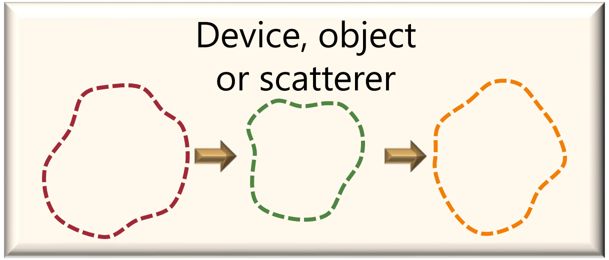
- "source" or input spaces
- and "receiving" or output spaces

They are "modes" in **two** spaces

not one space

They are **not** the "beams" between the spaces





"Waves, modes, communications, and optics: a tutorial," Adv. Opt. Photon. **11**, 679-825 (2019)

Communication modes

To set up the mathematics of this problem

we consider two spaces

A source or input volume V_S (rigorously, a Hilbert space H_S)

containing the possible source functions

written using a Dirac notation for convenience, e.g., $|\psi_S\rangle$

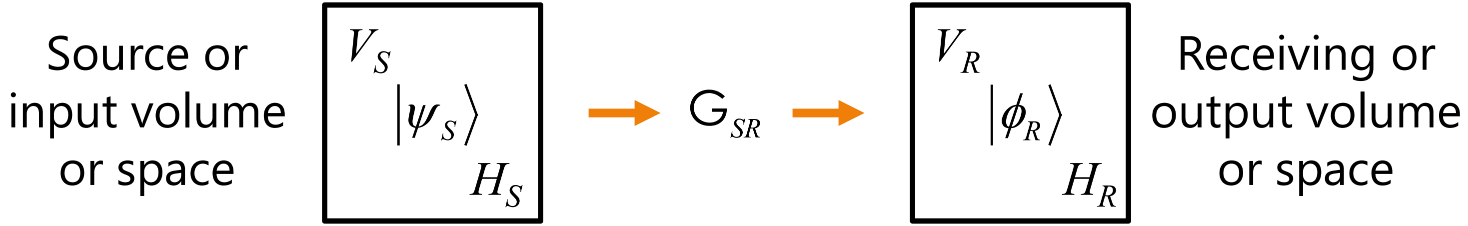
A receiving or output volume V_R (rigorously a Hilbert space H_R)

containing the possible wave functions

written using a Dirac notation for convenience, e.g., $|\phi_R\rangle$

Communication modes

The sources in the input space give waves in the receiving space through some coupling operator G_{SR}



For free space, this would be based on a free-space Green's function

such as a scalar monochromatic Green's function

$$G_\omega(\mathbf{r}_R;\mathbf{r}_S) = -\frac{1}{4\pi}\frac{\exp\left(ik\left|\mathbf{r}_R - \mathbf{r}_S\right|\right)}{\left|\mathbf{r}_R - \mathbf{r}_S\right|}$$

giving the wave at point $\mathbf{r}_R$ in the receiving space from the point source at $\mathbf{r}_S$ in the source space

Choosing eigen problems

We want eigen problems to get modes

- but we need *two* eigen problems because we have two different spaces

But these are *not* just the usual eigen problems of, say,

- a resonator in each volume

There is, however, a key mathematical trick we can use instead

Finding mode pairs

With the coupling operator G_{SR} between the spaces

- for the source space, solve the eigen problem for

  the operator $G_{SR}^{\dagger} G_{SR}$

$$G_{SR}^{\dagger} G_{SR} \left| \psi_{Sj} \right\rangle = \left| s_j \right|^2 \left| \psi_{Sj} \right\rangle$$

which gives an orthogonal set of

source functions (or mathematical vectors) $\left| \psi_{Sj} \right\rangle$

in H_S

Note: $G_{SR}^{\dagger}$ is the Hermitian adjoint of G_{SR}. As a matrix, it would be the complex conjugate of the transpose of the matrix. As a Green's function, it is the complex conjugate, with the roles of "source" and "receiver" points interchanged.

Finding mode pairs

With the coupling operator G_{SR} between the spaces

- for the receiving space, solve the eigen problem for the operator $G_{SR}G_{SR}^{\dagger}$

$$G_{SR}G_{SR}^{\dagger}\left|\phi_{Rj}\right\rangle = \left|s_j\right|^2\left|\phi_{Rj}\right\rangle$$

which gives an orthogonal set of wave functions (or mathematical vectors) $\left|\phi_{Rj}\right\rangle$ in H_R

(These problems have the same, positive eigenvalues $\left|s_j\right|^2$)

Finding mode pairs

When we have done this, we find that

$$G_{SR}\left|\psi_{Sj}\right\rangle = s_j \left|\phi_{Rj}\right\rangle$$

So, the source eigenfunction $\left|\psi_{Sj}\right\rangle$

generates the corresponding eigenfunction

the wave $\left|\phi_{Rj}\right\rangle$ in the receiving space

with the coupling amplitude s_j

We have established the communication mode **pairs** of functions

This process is the **singular-value decomposition** of the coupling operator G_{SR}

Matrix description of singular value decomposition (SVD)

For any linear operator D

at least as long as it is bounded, i.e., finite output for finite input

we can perform the singular value decomposition

$$\mathrm{D} = \mathrm{V}\mathrm{D}_{diag}\mathrm{U}^{\dagger} \quad \text{or equivalently} \quad \mathrm{D} = \sum_m s_m |\phi_m\rangle\langle\psi_m|$$

U and V are unitary operators ($\mathrm{U}^{\dagger}$ is automatically also unitary)

D_{diag} is a diagonal operator with elements s_m

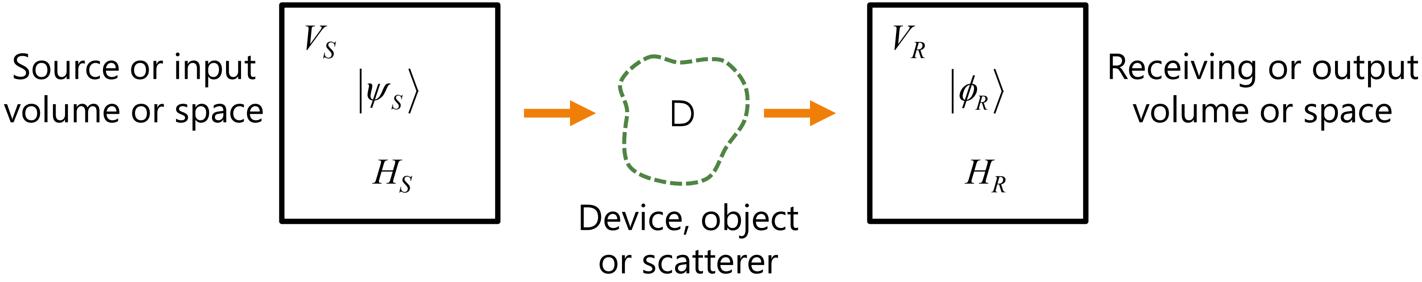
which are called the singular values

$|\psi_m\rangle$ are the columns of U (and $\langle\psi_m|$ are the rows of $\mathrm{U}^{\dagger}$)

$|\phi_m\rangle$ are the columns of V

Mode-converter basis sets

"All linear optical devices are mode converters," Opt. Express **20**, 23985 (2012)



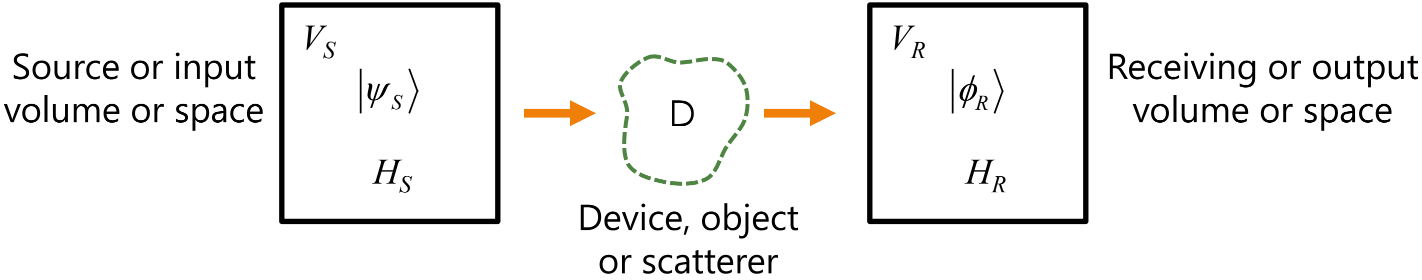
We can use singular-value decomposition (SVD)

for the more general case of a scatterer, optical device, or object

described by some operator D

Mode-converter basis sets

"All linear optical devices are mode converters," Opt. Express **20**, 23985 (2012)



One immediate consequence is that

because we can perform the SVD of any linear operator D

we have what we can call

the **mode-converter basis sets** of functions

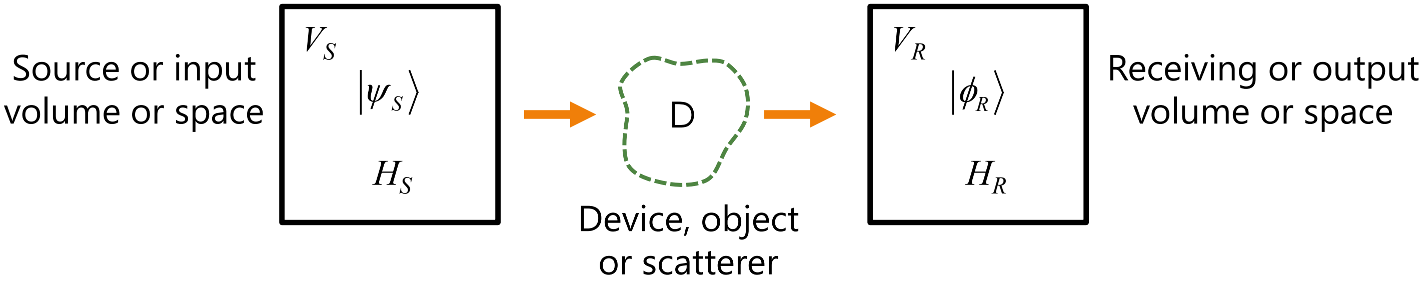
a set of orthogonal source functions

that lead, one by one

to a set of corresponding orthogonal received waves

Mode-converter basis sets

"All linear optical devices are mode converters," Opt. Express **20**, 23985 (2012)



In turn, that means that

there is a set of orthogonal channels through any linear scatterer

which are given by these mode-converter input and output function pairs

a generalization of the communication modes

now used as a description of an optical device or scatterer

Communication mode and mode converter basis sets

This realization that any optical system can be represented using

an orthogonal set of input functions

that map, one by one

to an orthogonal set of output functions

is quite a profound one in optics

It leads to new fundamental results

and a new way of making arbitrary optics

"Communicating with Waves Between Volumes – Evaluating Orthogonal Spatial Channels and Limits on Coupling Strengths," Appl. Opt. **39**, 1681 (2000).

"All linear optical devices are mode converters," Opt. Express **20**, 23985-23993 (2012)

"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679 (2019)

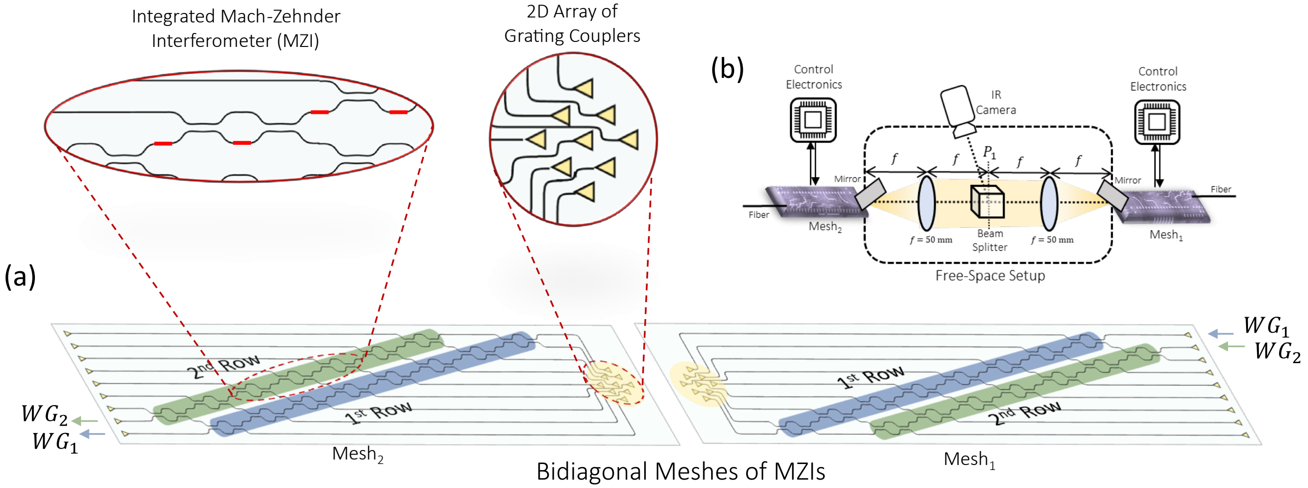
Establishing optimum orthogonal channels - experiment

This apparatus we discussed above

with two self-configuring interferometer meshes

finds the first two communication modes through this optical system

by a process of maximization, without calculations



S. SeyedinNavadeh, M. Milanizadeh, F. Zanetto, G. Ferrari, M. Sampietro, M. Sorel, D. A. B. Miller, A. Melloni, and F. Morichetti, "Determining the optimal communication channels of arbitrary optical systems using integrated photonic processors," Nat. Photon. **18**, 149-155 (2024)

“Flipping round” the SVD

Now, we know that we can construct any unitary linear operator in optics

- using a mesh of interferometers

And we now know we can perform the SVD of any linear optical system

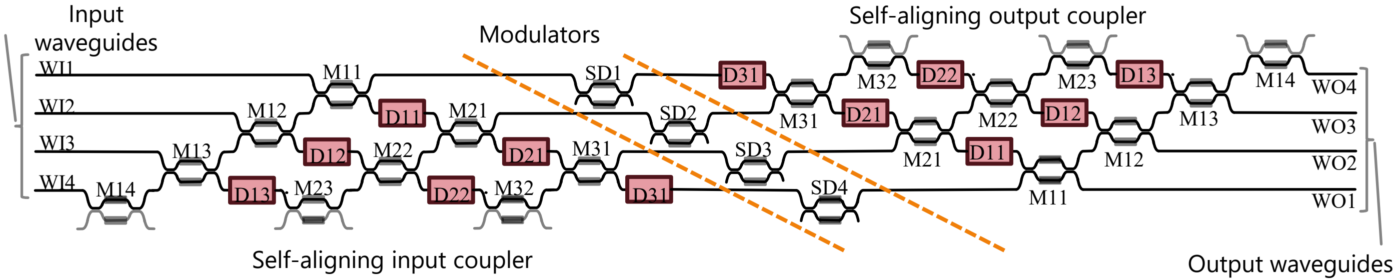
- which decomposes it mathematically into a product of three operators
  - a unitary, a diagonal and a unitary

Can we take one more step

- and emulate *any* linear operator with interferometer meshes?

General multiple mode converter

"Self-configuring universal linear optical component," Photon. Res. **1**, 1 (2013)



The self-aligning input coupler mesh on the left can couple any four orthogonal inputs

each to different single waveguides in the middle

This is a first, arbitrary "unitary matrix" multiplication

The amplitude and phase of this conversion can be controlled by the modulators in the middle

These modulators are implementing the singular values

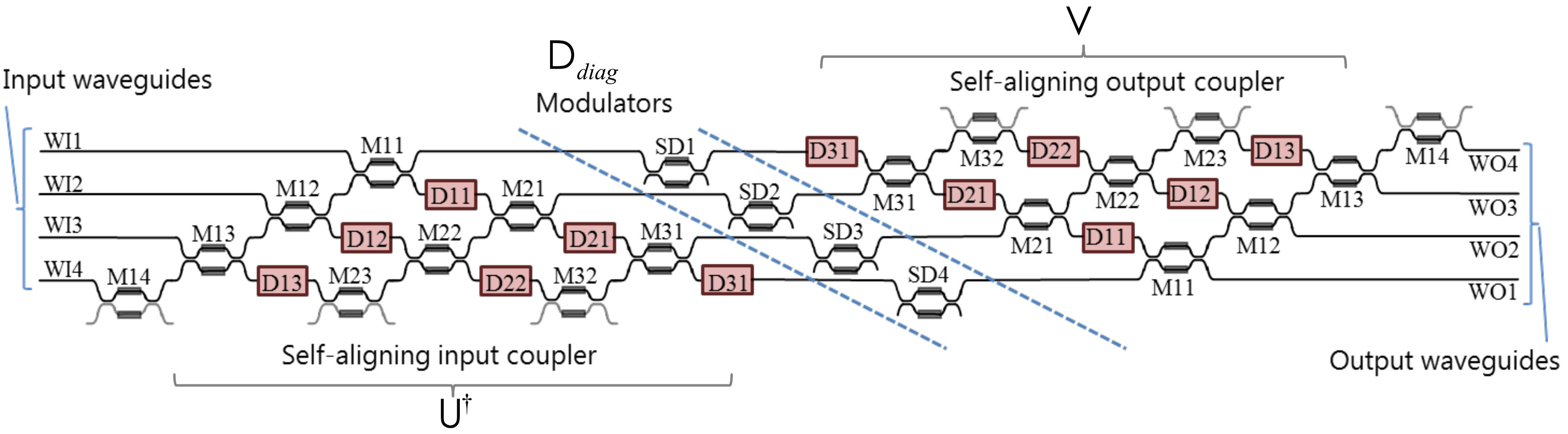
Light in those single waveguides can be converted into any other set of four orthogonal outputs on the right

by the self-aligning output coupler mesh on the right

This is the second arbitrary "unitary matrix" multiplication

General multiple mode converter

"Self-configuring universal linear optical component," Photon. Res. **1**, 1 (2013)



So, the optical "units" in the mesh implement the singular value decomposition $D = VD_{diag}U^\dagger$

So, for an optical system of a given dimensionality

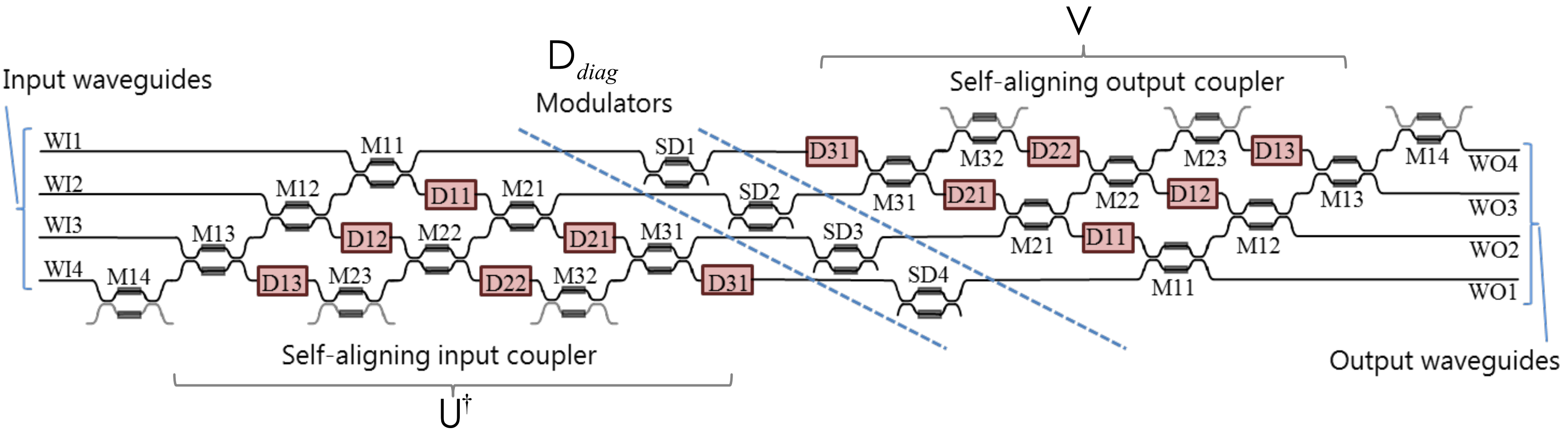
we can emulate any linear optical system

Note we are implementing an arbitrary linear optical component

by constructing it using its mode converter basis sets

General multiple mode converter

"Self-configuring universal linear optical component," Photon. Res. **1**, 1 (2013)



The input mode converter basis functions are the ones that

are converted to light in single waveguides in the middle

The output mode converter basis functions are the ones

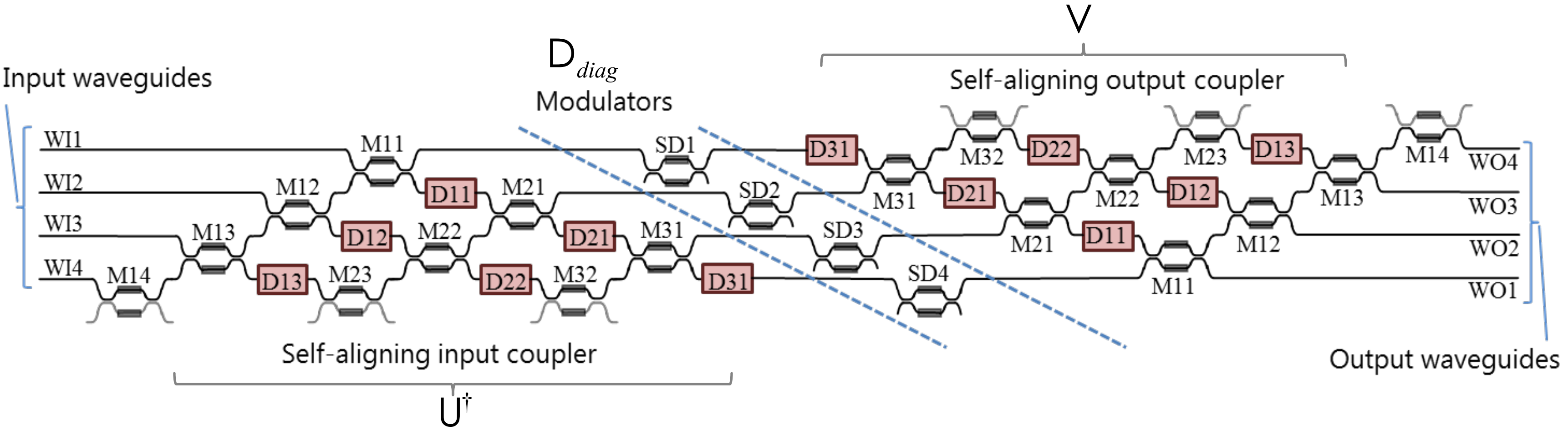
generated by light in a single waveguide in the middle

The coupling strengths from input to output mode-converter modes

are the singular values implemented by the modulators in the middle

General multiple mode converter

"Self-configuring universal linear optical component," Photon. Res. **1**, 1 (2013)



This is the first proof that any linear optical component is possible in principle

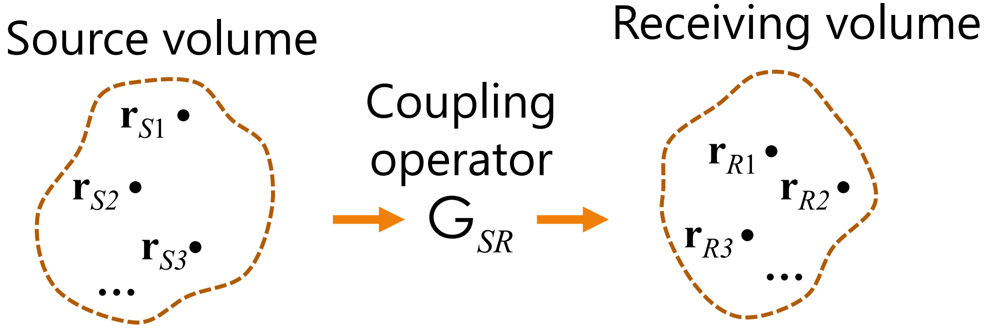
and that any linear optical system can be factored into a set of 2-beam interferences

The proof is that we have shown how you can make it

Synthesizing wave fields

stanford.io/47BsEiw

Constructing examples with point sources and receivers



We can see how this works first for a finite number of point sources and receivers

- e.g., "loudspeakers" at positions $\mathbf{r}_{S1}$, $\mathbf{r}_{S2}$, $\mathbf{r}_{S3}$, etc., in the source volume
- and "microphones" at positions $\mathbf{r}_{R1}$, $\mathbf{r}_{R2}$, $\mathbf{r}_{R3}$, etc., in the receiving volume

Using the Green's function, we can construct the resulting matrix to represent G_{SR}

3 sources and receivers

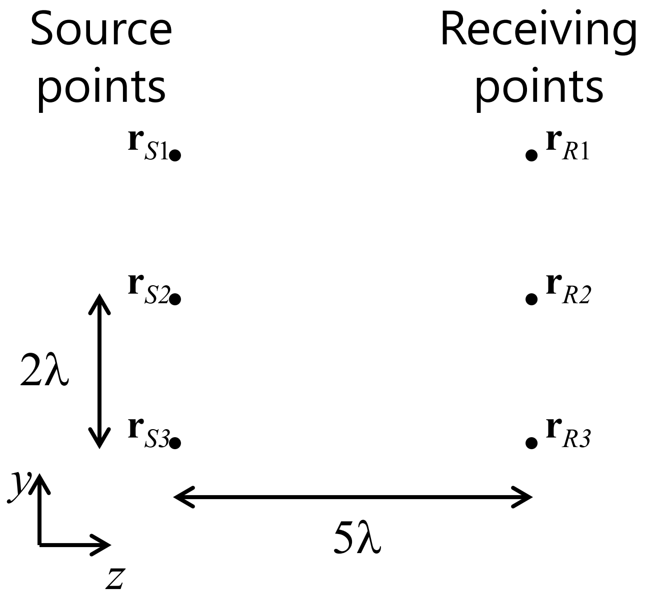
For these source and receiving points

using the Green's function

$$G_\omega(\mathbf{r}_R;\mathbf{r}_S) = -\frac{1}{4\pi}\frac{\exp\left(ik\left|\mathbf{r}_R - \mathbf{r}_S\right|\right)}{\left|\mathbf{r}_R - \mathbf{r}_S\right|}$$

gives the matrix (for unit wavelength λ)

$$\mathrm{G}_{SR} \cong \frac{-1}{62.83}\begin{bmatrix} 1 & -0.7+0.6i & -0.64+0.45i \\ -0.7+0.6i & 1 & -0.7+0.6i \\ -0.64+0.45i & -0.7+0.6i & 1 \end{bmatrix}$$



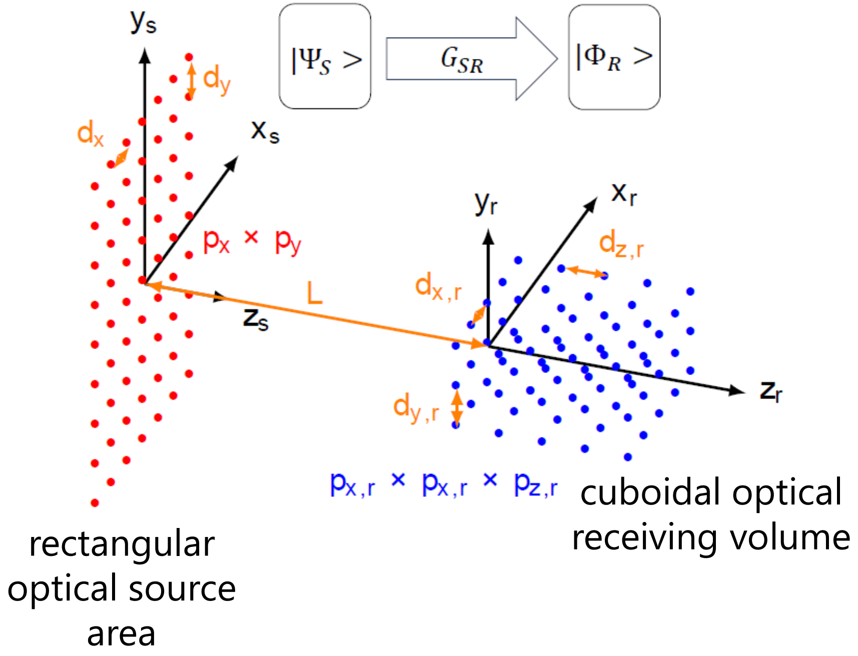
A larger example

Dense sets of points in

- a planar rectangular optical source "volume"
- a cuboidal optical receiving volume

We can similarly construct the matrix representing G_{SR}

- and perform the SVD of it
  - to get the source and receiving vectors and coupling strengths (singular values)



V. S. de Angelis, A. H. Dorrah, L. A. Ambrosio, D. A. B. Miller, and F. Capasso, "3D holography using communication mode optics," in *Optica Imaging Congress 2024 (3D, AOMS, COSI, ISA, pcAOP)*, Technical Digest Series (Optica Publishing Group, 2024), paper DF4H.4. https://doi.org/10.1364/3D.2024.DF4H.4

Generating arbitrary waves with communication modes

Suppose we want a specific wave $|\phi_{Ro}\rangle$ in the receiving space

We expand it in the "receiving" communication modes as $|\phi_{Ro}\rangle = \sum_j a_j |\phi_{Rj}\rangle$

where $a_j = \langle \phi_{Rj} | \phi_{Ro} \rangle$ is the "inner product" or "overlap integral"

Since $G_{SR} |\psi_{Sj}\rangle = s_j |\phi_{Rj}\rangle$

to generate any specific component $a_q |\phi_{Rq}\rangle$ for this expansion

we need an amplitude a_q / s_q of the source function $|\psi_{Sq}\rangle$

So, the required source function $|\psi_{So}\rangle$ to generate $|\phi_{Ro}\rangle$ is

$$|\psi_{So}\rangle = \sum_j \frac{a_j}{s_j} |\psi_{Sj}\rangle \equiv \sum_j \frac{1}{s_j} \langle \phi_{Rj} | \phi_{Ro} \rangle |\psi_{Sj}\rangle$$

This lets us generate any desired wave in the receiving space

even if the coupling strengths s_j are not the same for every communication mode

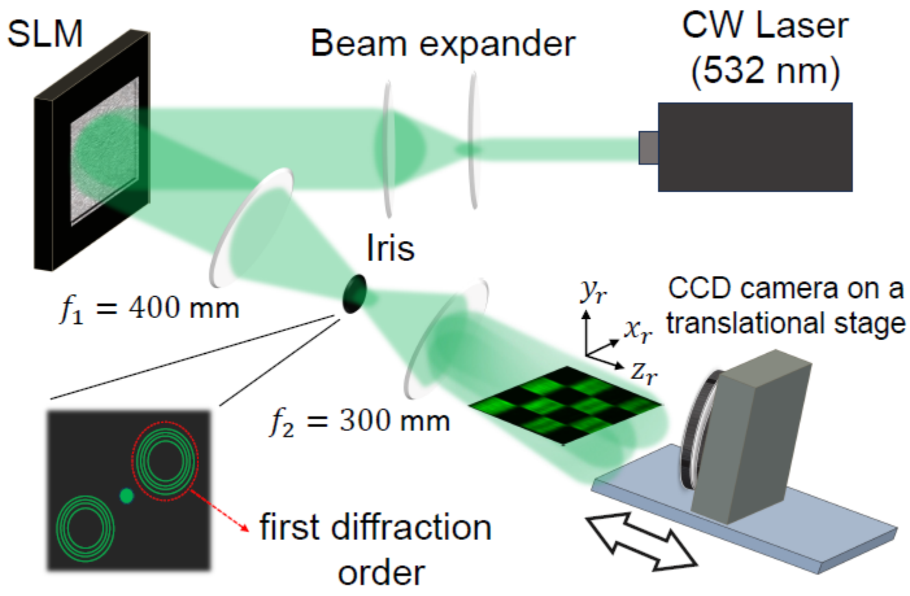
"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679 (2019)

de Angelis et al., "3D holography using communication mode optics," in *Optica Imaging Congress 2024*, paper DF4H.4. https://doi.org/10.1364/3D.2024.DF4H.4

Designing and creating a 3-D optical field

Using this approach, we can generate an arbitrary desired 3D field

- calculating the necessary amplitudes for the pixel “point sources” on a spatial light modulator
  - to generate the field of interest in the 3D volume



de Angelis et al., "3D holography using communication mode optics," in *Optica Imaging Congress 2024*, paper DF4H.4. https://doi.org/10.1364/3D.2024.DF4H.4

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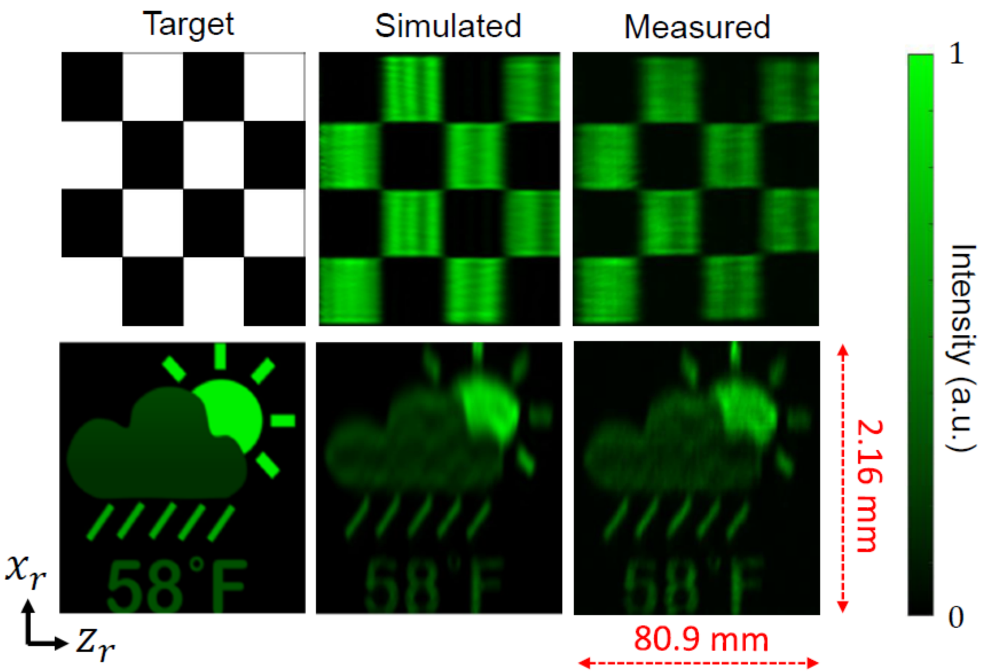

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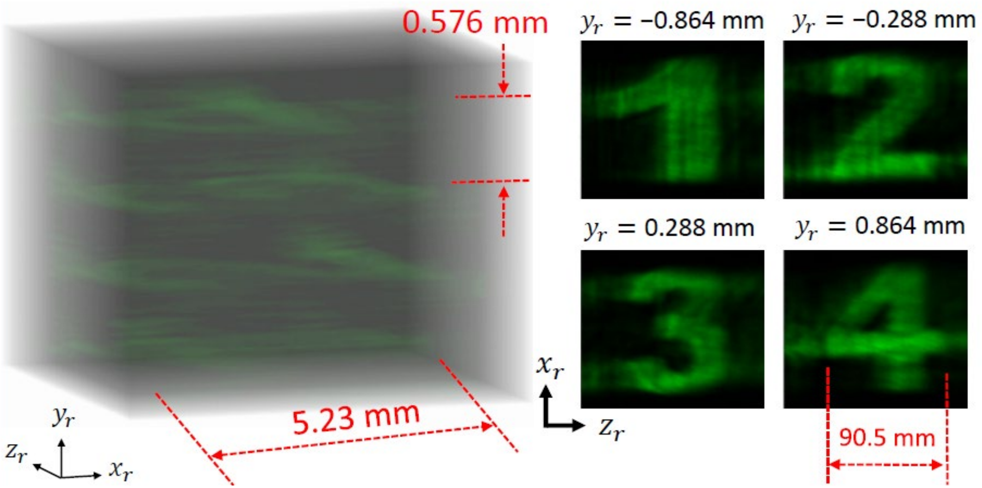


de Angelis et al., "3D holography using communication mode optics," in *Optica Imaging Congress 2024*, paper DF4H.4. https://doi.org/10.1364/3D.2024.DF4H.4

Waves, modes, and minimum thicknesses for optics

stanford.io/47BsEiw

DM, "Why optics needs thickness," Science 379, 41 (2023)

Why optics needs thickness

For metasurfaces and metastructures

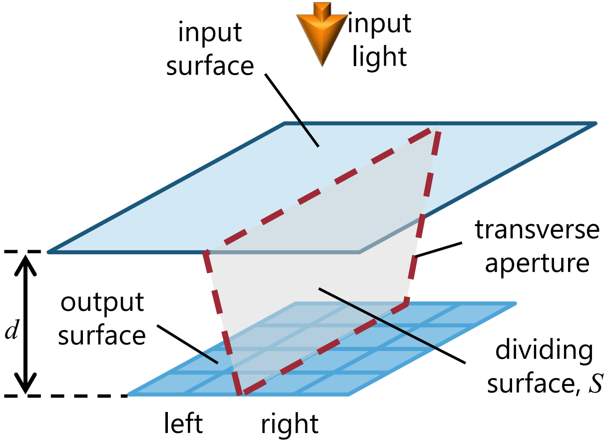
and for compact optics generally

we need to understand whether they need thickness

Can we make a given optical device in just one "layer", for example?

Generally, no.

But why?

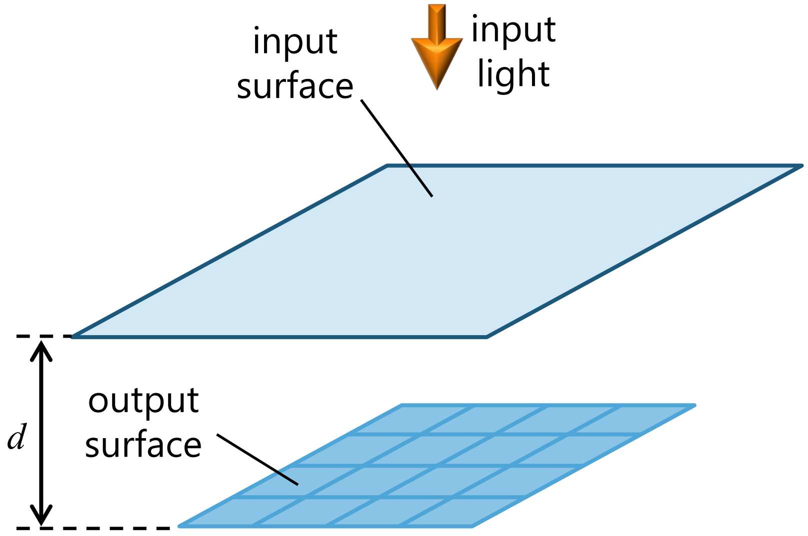


David Miller, "Why optics needs thickness," Science 379, 41 (2023)

Why optics needs thickness

Think of an optical system with
- an input surface
  - such as a lens surface or metasurface
- an output surface
  - such as an image sensor plane
- with a distance *d* between them

Note we are not yet specifying what is between these two surfaces
- and we will not need to do so



DM, Science 379, 41 (2023)

The key idea – channels through a transverse aperture

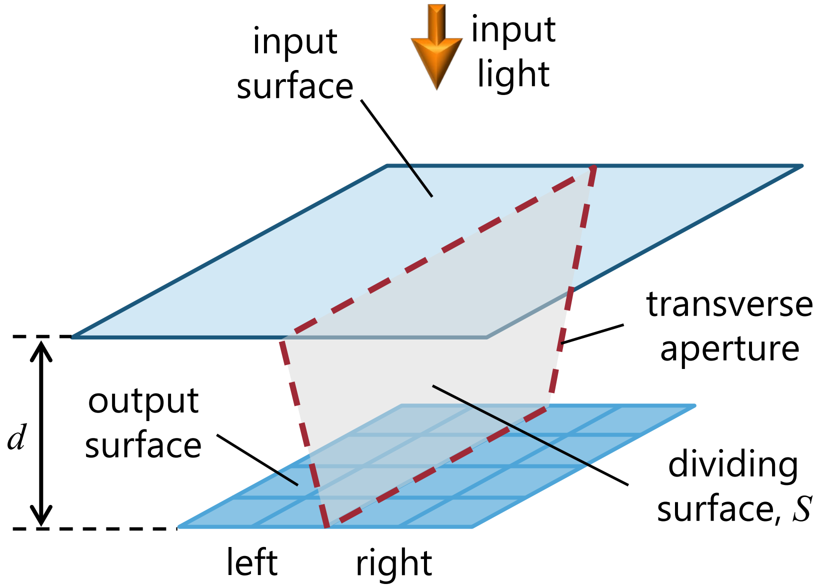
Now imagine we divide each surface in two parts
- left and right
  - by passing an imaginary mathematical dividing surface *S* through them

This defines a "**transverse aperture**"

Because of what we want the system to do
- some number *C* of channels must pass
  - from right to left (or left to right)
    - through this aperture

We call *C* the "**overlapping nonlocality**"

The transverse aperture must be large enough
- for these channels to propagate through it
  - which requires minimum area and/or thickness



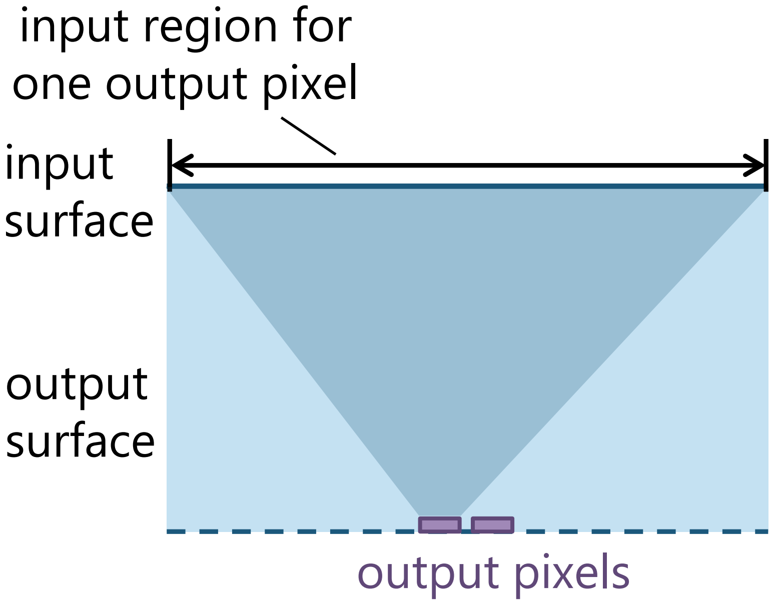
DM, Science 379, 41 (2023)

Nonlocality in optics

nonlocality

the output at one point depends on the input at many points

Imager example



Nonlocality in optics

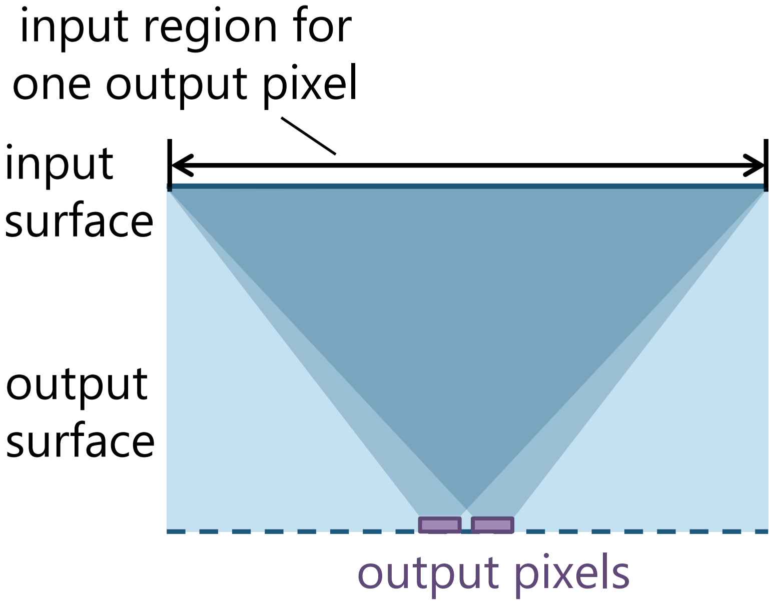
nonlocality

- the output at one point depends on the input at many points

overlapping nonlocality

- the input regions for different output points overlap with one another

Imager example



Nonlocality in optics

nonlocality

the output at one point depends on the input at many points

overlapping nonlocality

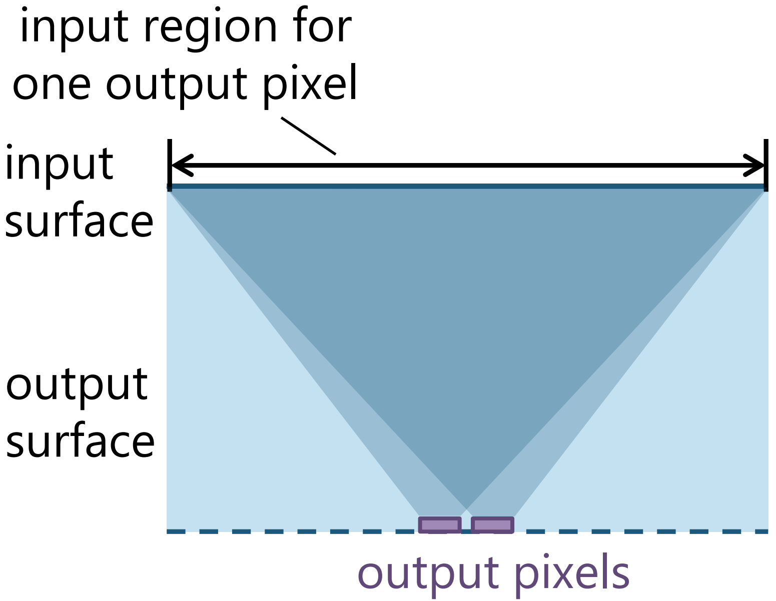
the input regions for different output points overlap with one another

overlapping nonlocality C

loosely, the number of such overlapping "channels"

For an imager, C ends up being half the number of pixels

Imager example



Nonlocality in optics

nonlocality

- the output at one point depends on the input at many points

overlapping nonlocality

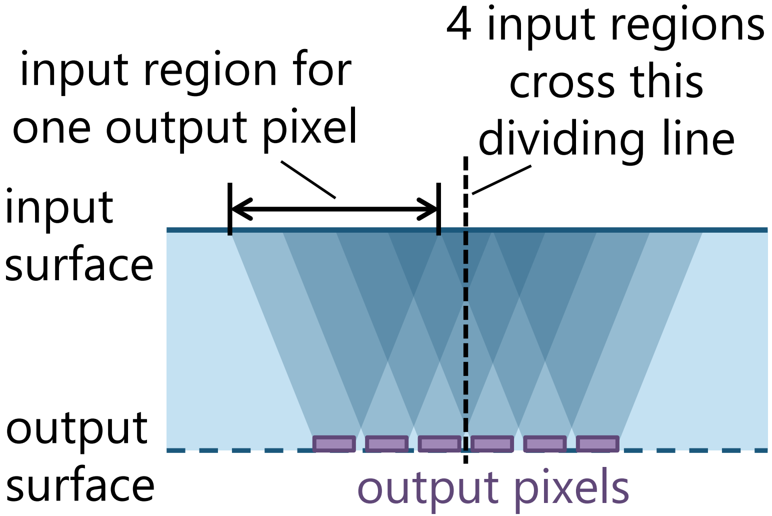
- the input regions for different output points overlap with one another

overlapping nonlocality C

- loosely, the number of such overlapping “channels”

For this example, C is 4

Space-invariant example
e.g., image differentiator



$C = 4$

How big a transverse aperture for a given C?

For a 1D system with free-space wavelength λ_o and maximum refractive index n_{max}

we presume we need a thickness

$$\Delta d \geq \lambda_o / 2\alpha n_{max}$$

for each channel

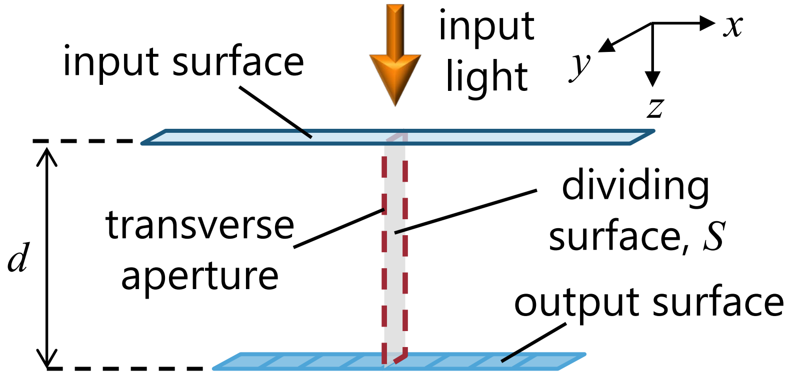
where we allow for some practical factor $\alpha > 1$

which comes from some practical restriction on the range of

usable angles

or usable k-space

inside the device



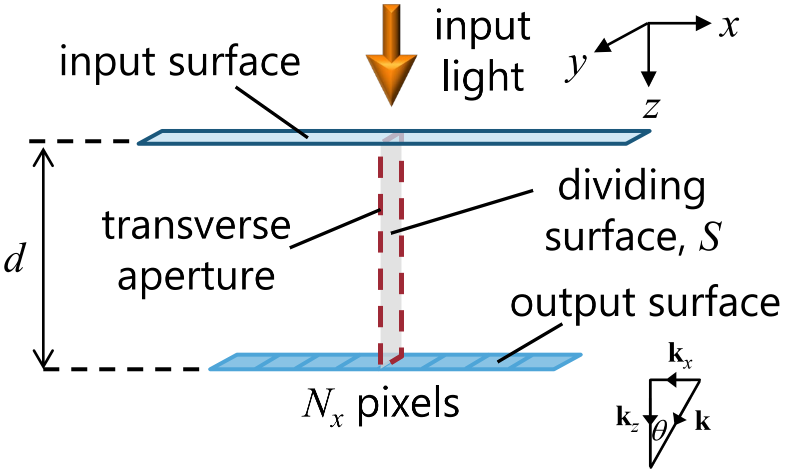
Thickness of a one-dimensional imager

Quite generally, for some value of C_x in a one-dimensional device

with $\Delta d \geq \lambda_o / 2\alpha n_{max}$ of thickness per channel

then $d \geq C_x \Delta d$

so $d \geq C_x \lambda_o / 2\alpha n_{max}$



For our one-dimensional imager with $C_x = N_x / 2$

$$d \geq N_x \lambda_o / 4\alpha n_{max}$$

For $\lambda_o = 700$ nm, $N_x = 4000$ (one "line" of a 12 MP smartphone camera), $n_{max} = 1.5$ and no "rays" past 45° angle, $d \geq 1.6$ mm

Note: this will also be the limit for a 2D imager based on conventional lenses

See V. Blahnik, O. Schindelbeck, Advanced Optical Technologies 10, 145 (2021) for general discussion of modern smartphone cameras

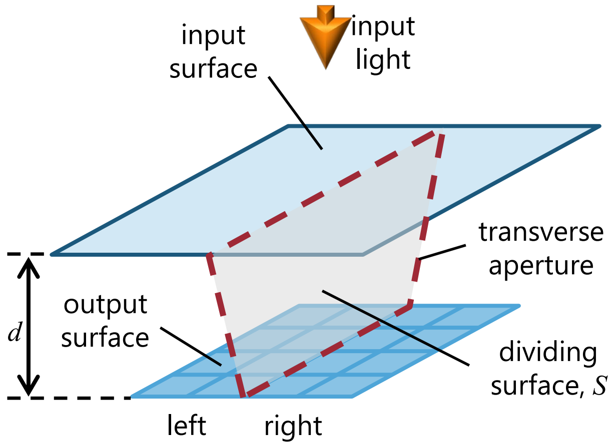
Why optics needs thickness

For a conventional cell phone camera

- in practice, even if we took all the thickness out of the lenses themselves
  - the camera would still need to be ~ 1.6 mm thick

The formal way to analyze this problem is to perform

- the singular value decomposition of the coupling between
  - the left side of the input and the right side of the output
    - allowing us to count the number of modes we need



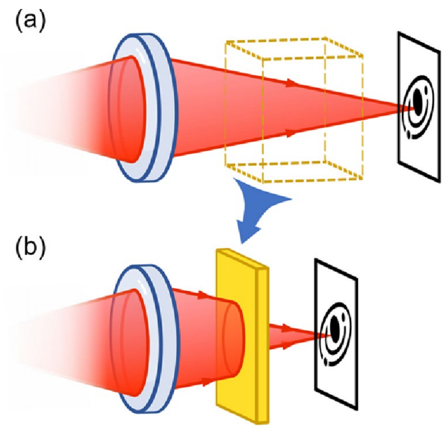
David Miller, "Why optics needs thickness," Science 379, 41 (2023)

Why optics needs thickness

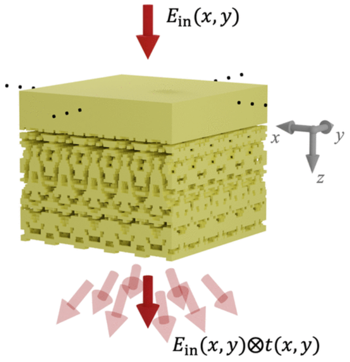
This explains the necessary thickness of, e.g.,

- smart phone cameras, which are within a factor of 3 of this limit
- “space plates” intended to make imagers thinner
- metasurface/metastructure devices as the “kernel” becomes more nonlocal

  e.g., as in image differentiation



Guo et al., Optica 7, 1133 (2020)



H. Wang et al., ACS Photonics 9, 1358 (2022)

David Miller, "Why optics needs thickness," Science 379, 41 (2023)

How many waves can get out of a volume?

stanford.io/47BsEiw

David Miller, *Stanford University*
Zeyu Kuang, Owen Miller, *Yale University*

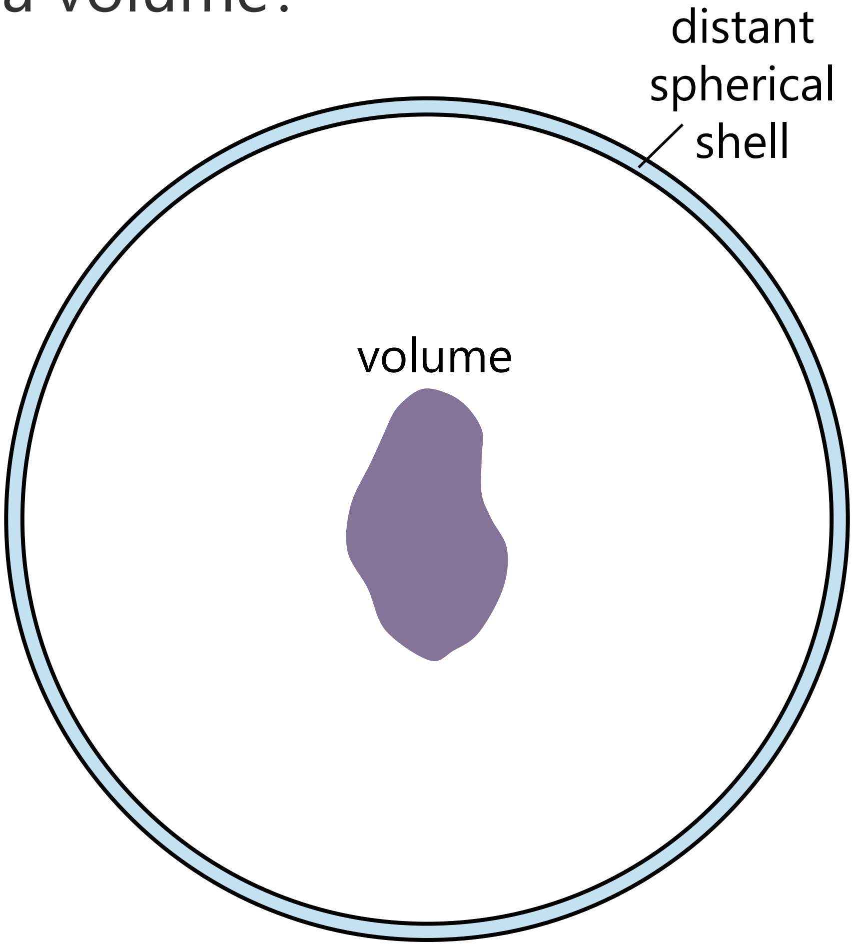
How many waves can get out of a volume?

Suppose we have some arbitrary volume

- which could contain
  - some optical source
    - or some set of antenna elements

Can we deduce just how many waves or channels

- can effectively get out of it
  - propagating into the far field
    - e.g., to a distant spherical shell?



The rigorous approach to channels between volumes

We return to the the singular-value decomposition of the coupling operator G_{SR}

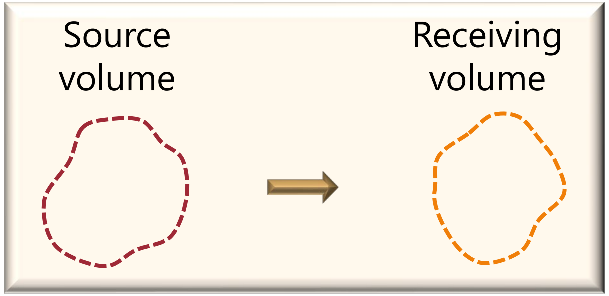
giving orthogonal source functions $|\psi_{Sj}\rangle$

that couple, one by one, to orthogonal received waves $|\phi_{Rj}\rangle$

with some coupling strength s_j

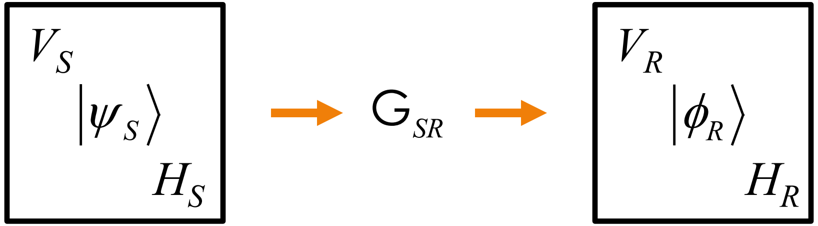
These pairs of functions $|\psi_{Sj}\rangle$ and $|\phi_{Rj}\rangle$

are the "communication modes"



"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679-825 (2019)

Source or input volume V_S or space H_S

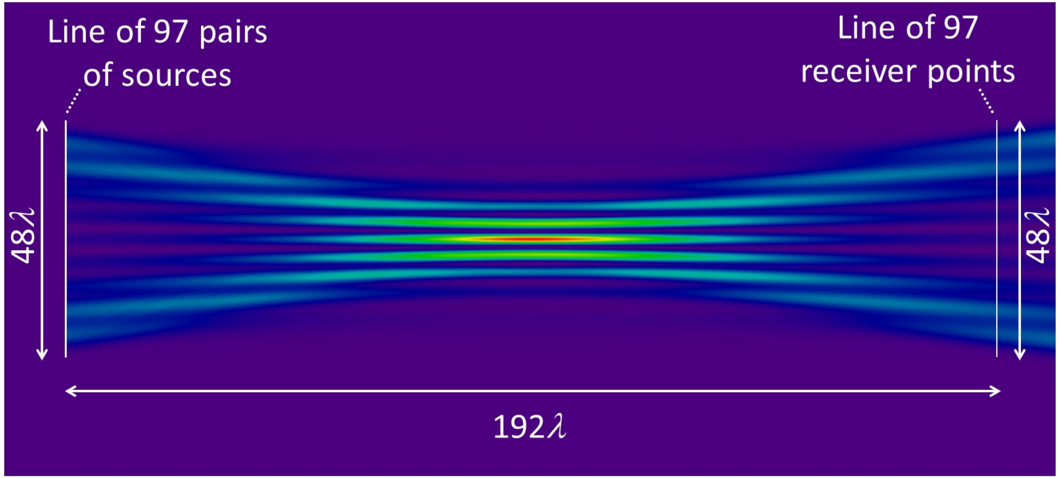


Receiving or output volume V_R or space H_R

A paraxial example

Suppose we have a line of sources and a line of receiver points

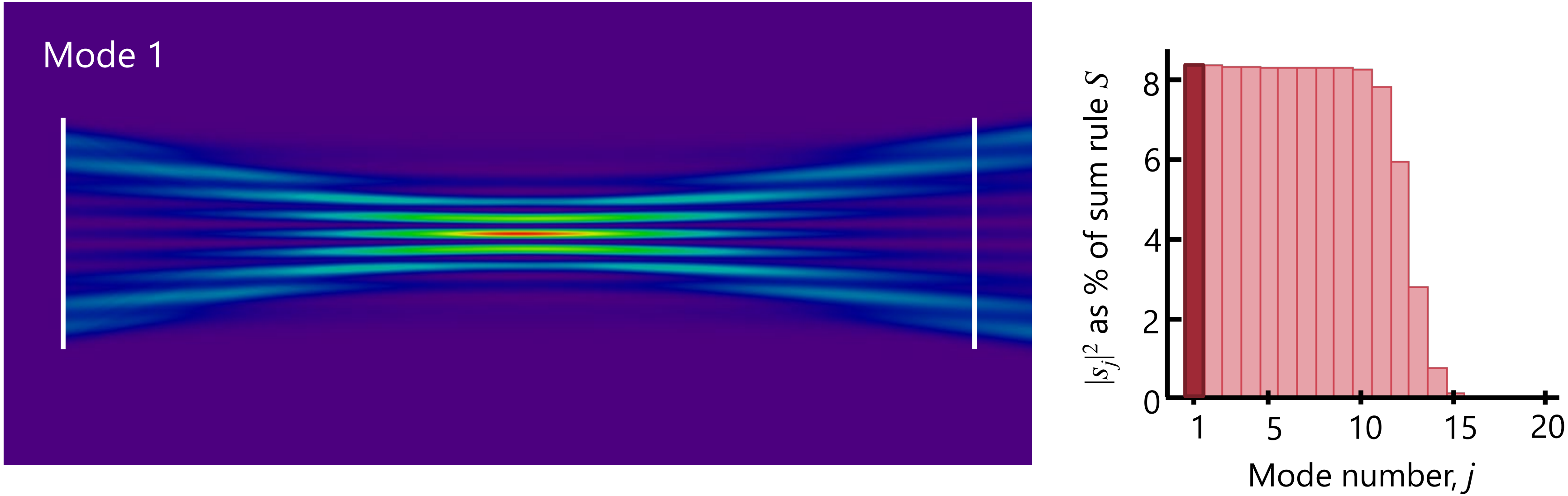
here in an approximately "paraxial" set of dimensions



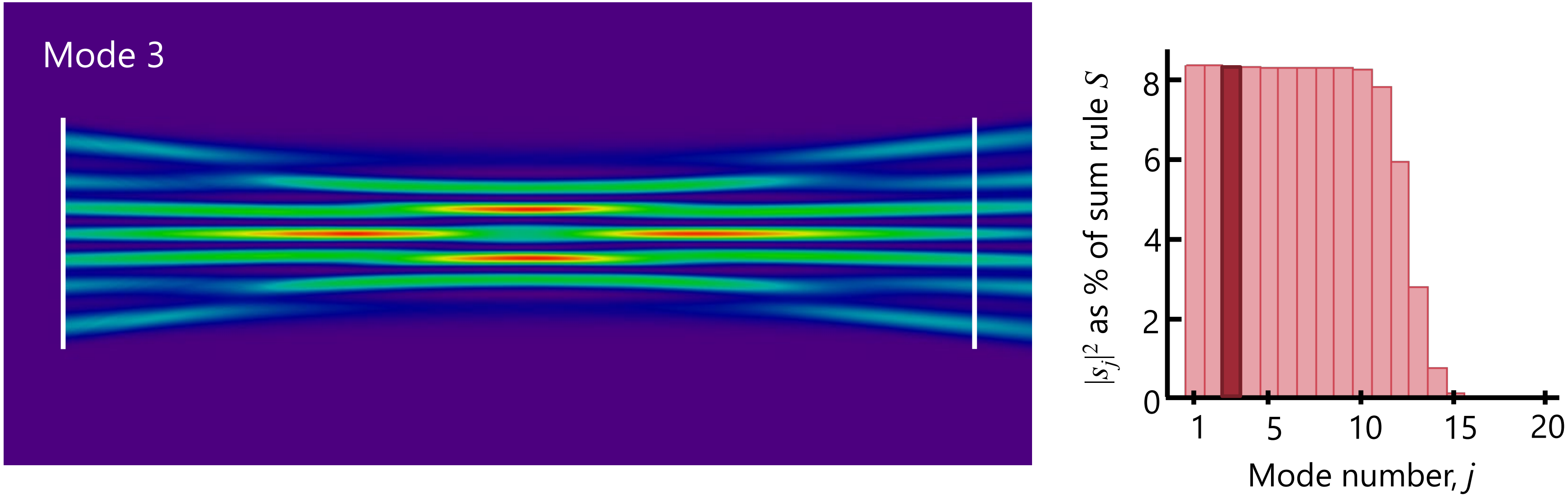
and we establish the communication modes between them

The picture shows the cross-section of the intensity in the plane

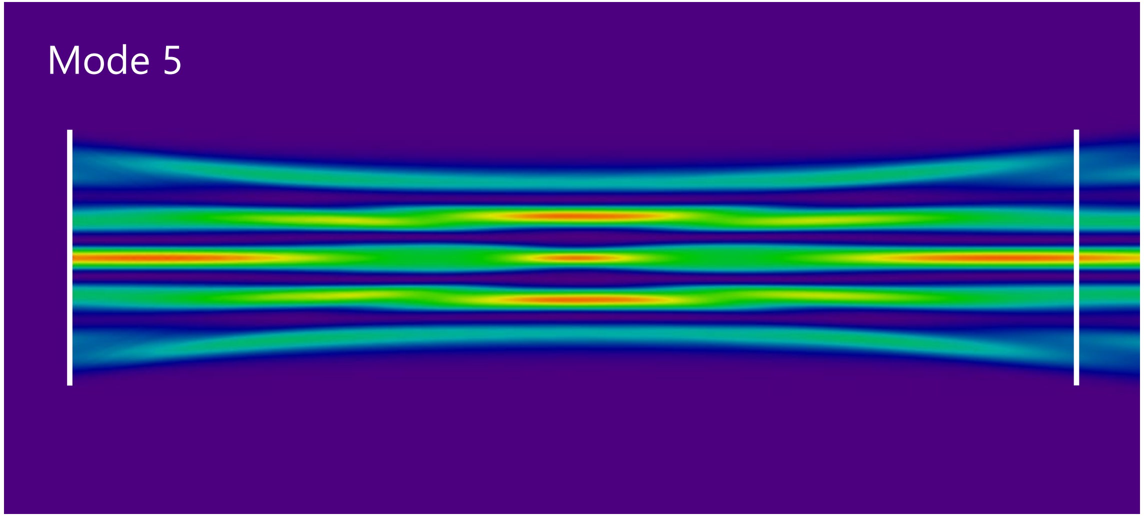
here for the most strongly coupled mode

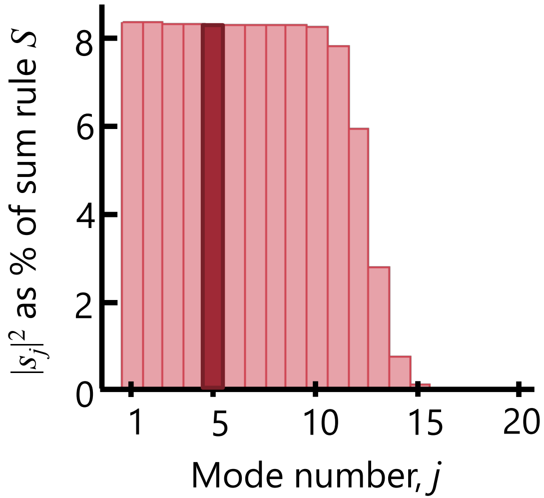


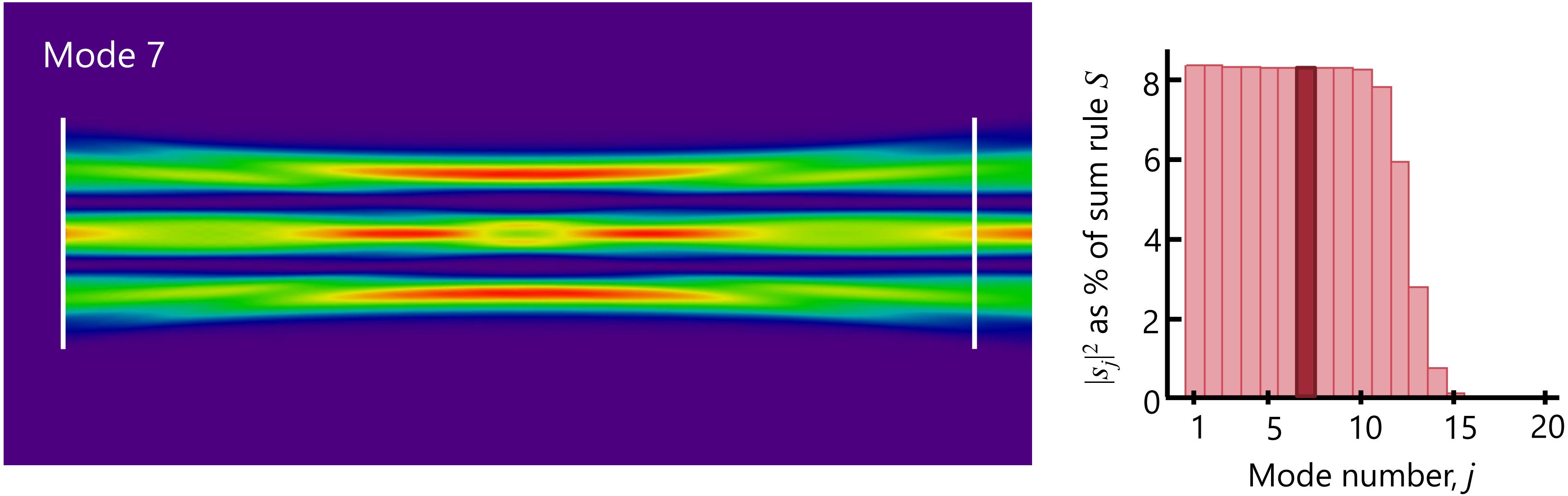
Mode 1
$|s_j|^2$ as % of sum rule S
0
2
4
6
8
1
5
10
15
20
Mode number, j



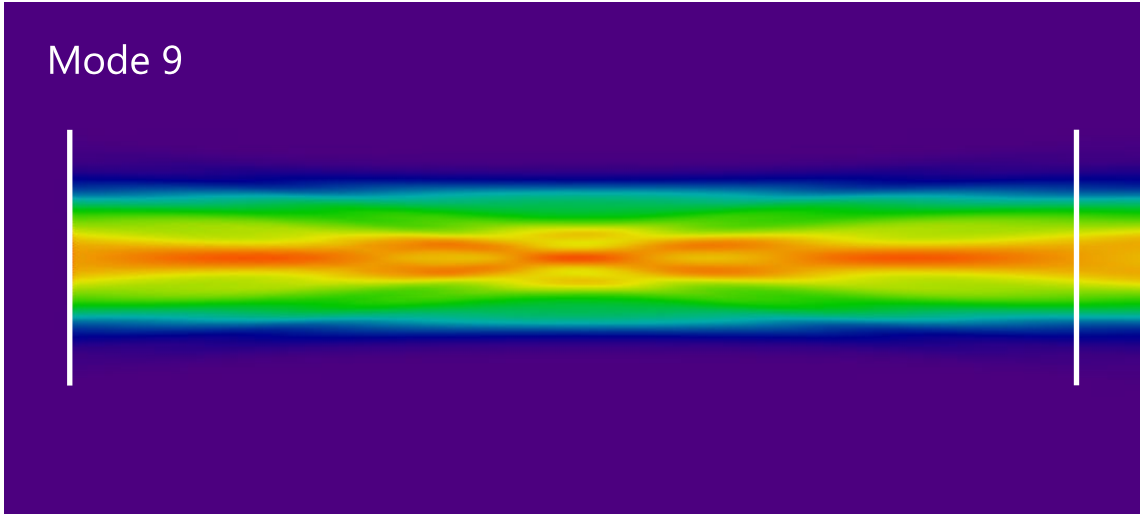
Mode 3
$|s_j|^2$ as % of sum rule S
0
2
4
6
8
1
5
10
15
20
Mode number, j


Mode 5

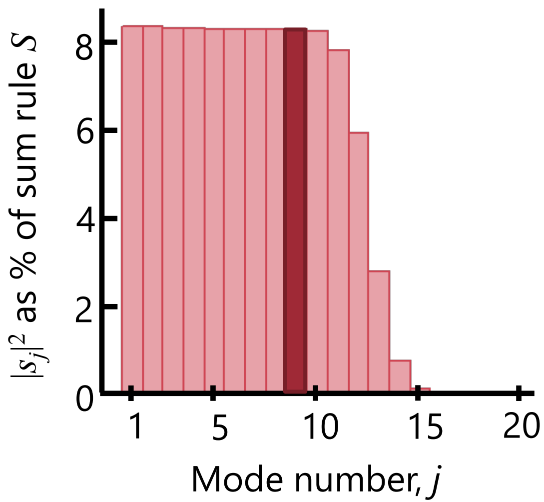

$|s_j|^2$ as % of sum rule S
0
2
4
6
8
1
5
10
15
20
Mode number, j



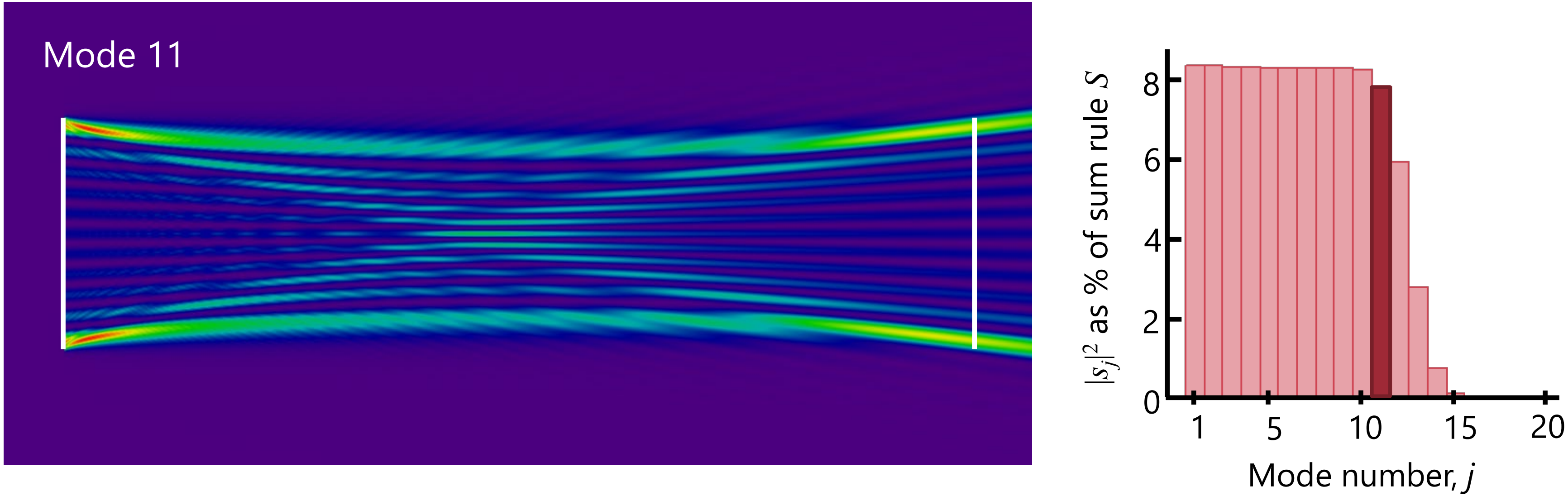
Mode 7
$|s_j|^2$ as % of sum rule S
8
6
4
2
0
1
5
10
15
20
Mode number, j



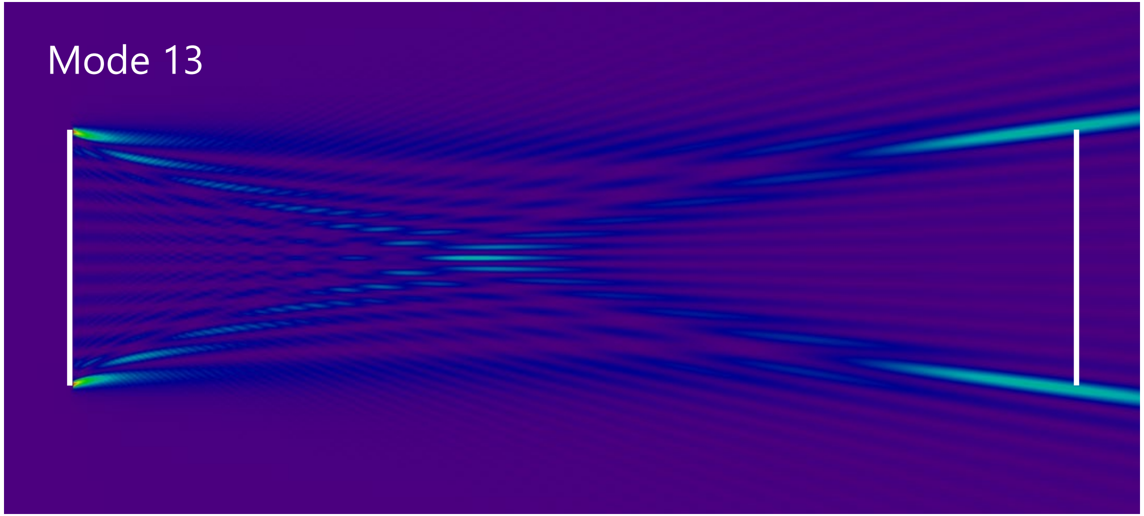
Mode 9



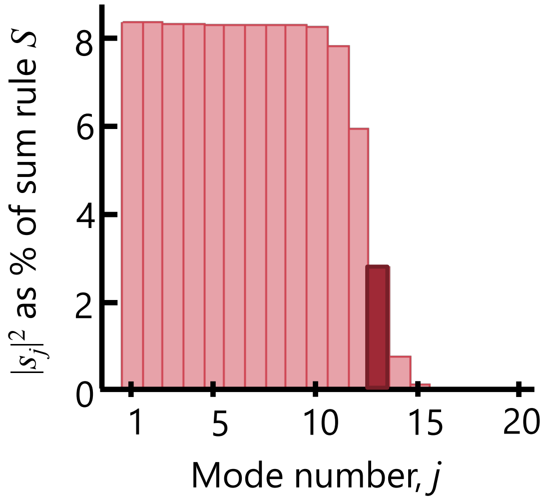
$|s_j|^2$ as % of sum rule S
0
2
4
6
8
1
5
10
15
20
Mode number, j



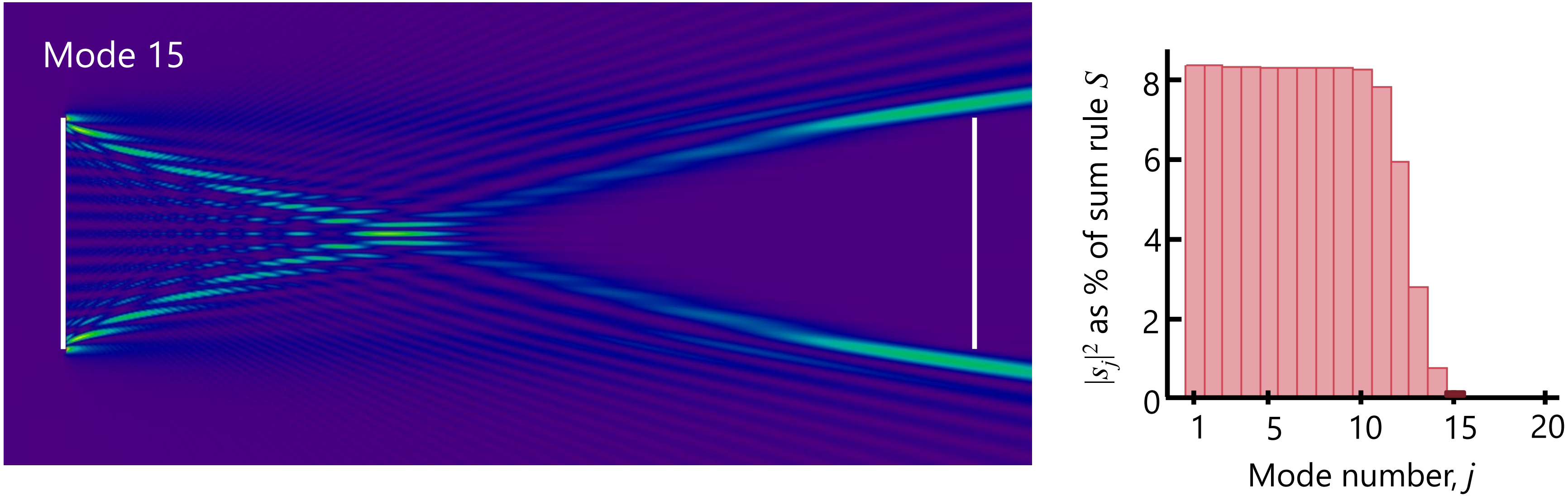
Mode 11
8
6
4
2
0
1
5
10
15
20
Mode number, j
|s_j|^2 as % of sum rule S

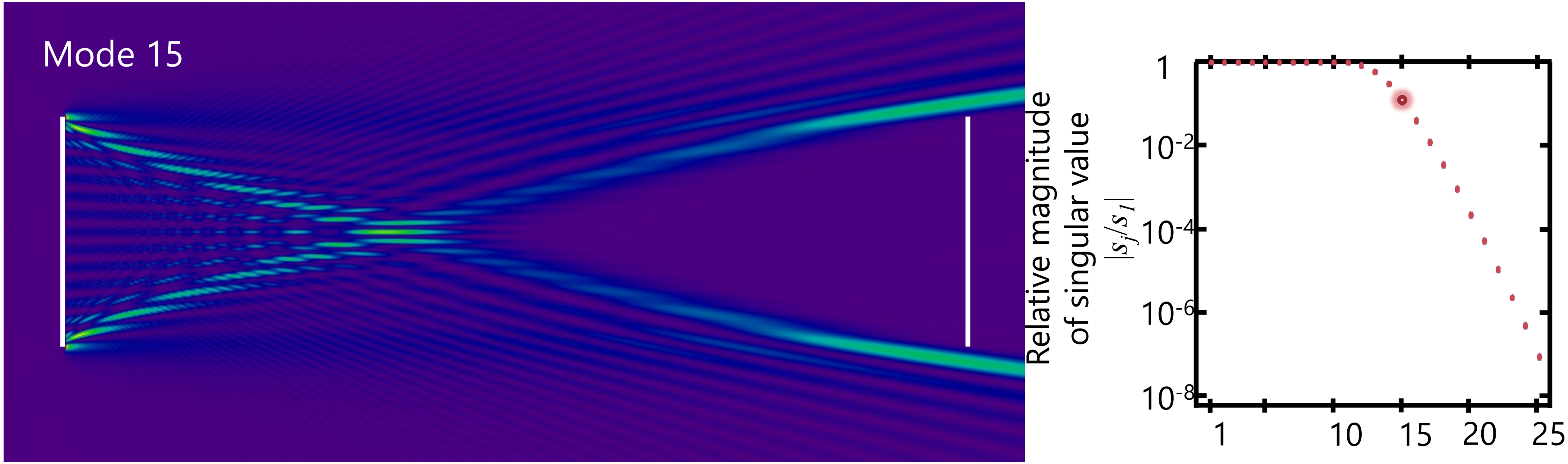


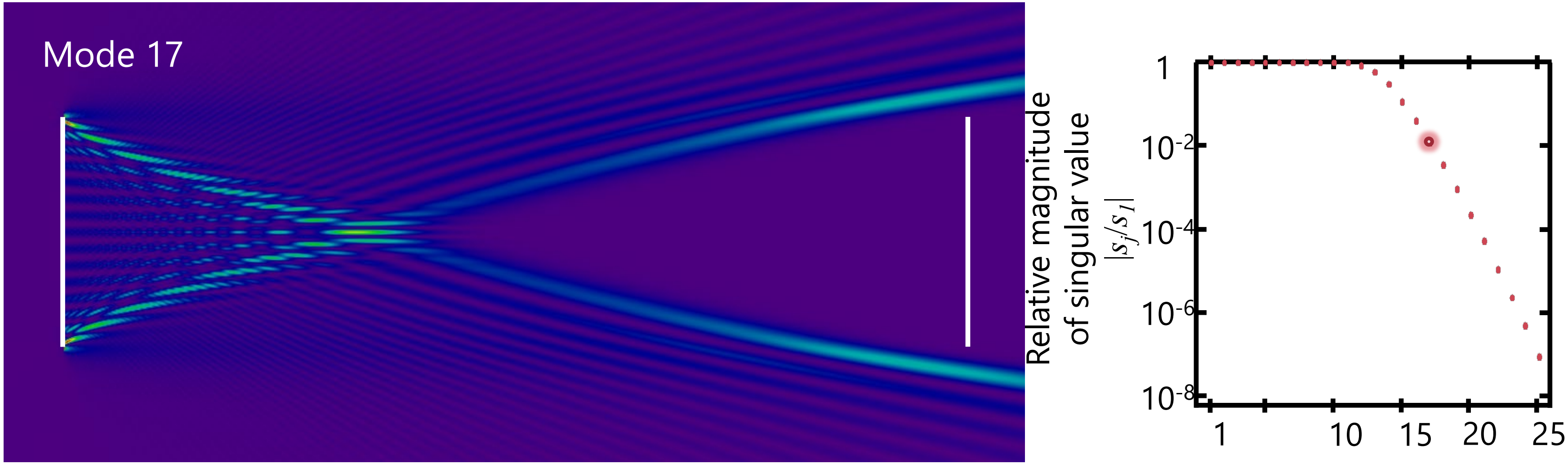
Mode 13



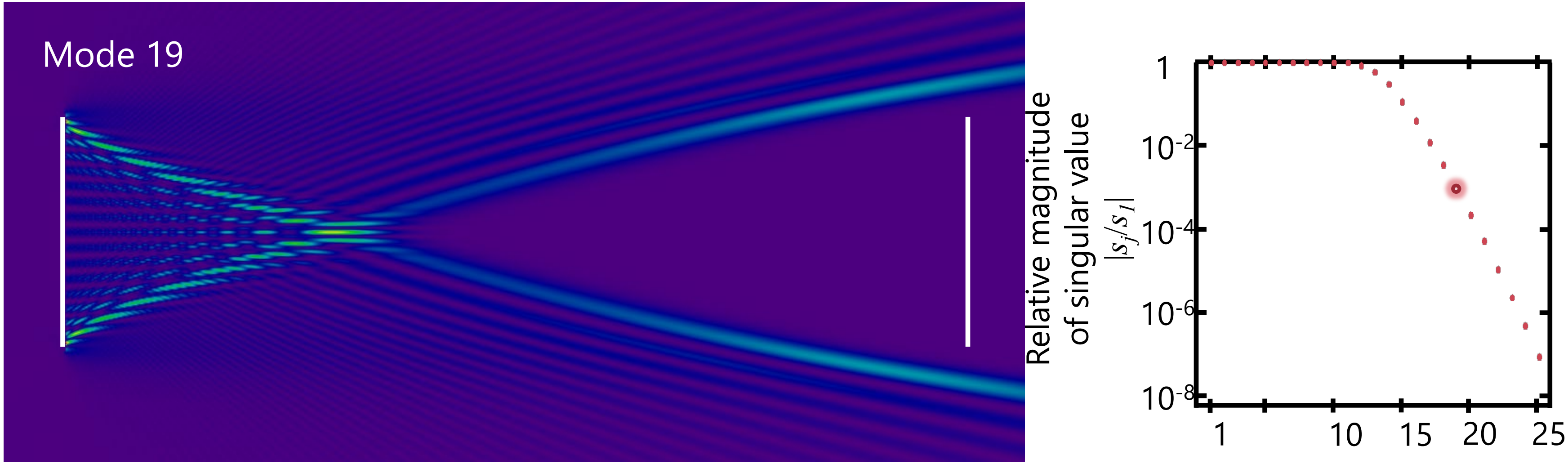
$|s_j|^2$ as % of sum rule S
8
6
4
2
0
1
5
10
15
20
Mode number, j


Mode 15
$|s_j|^2$ as % of sum rule S
0
2
4
6
8
1
5
10
15
20
Mode number, j


Mode 15
Relative magnitude of singular value
$|s_j/s_1|$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
15
20
25

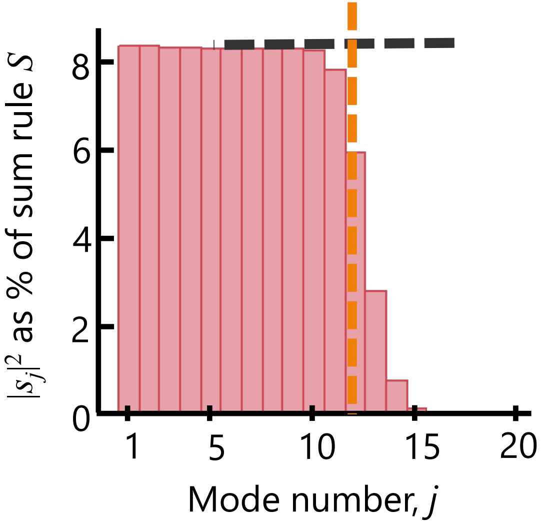


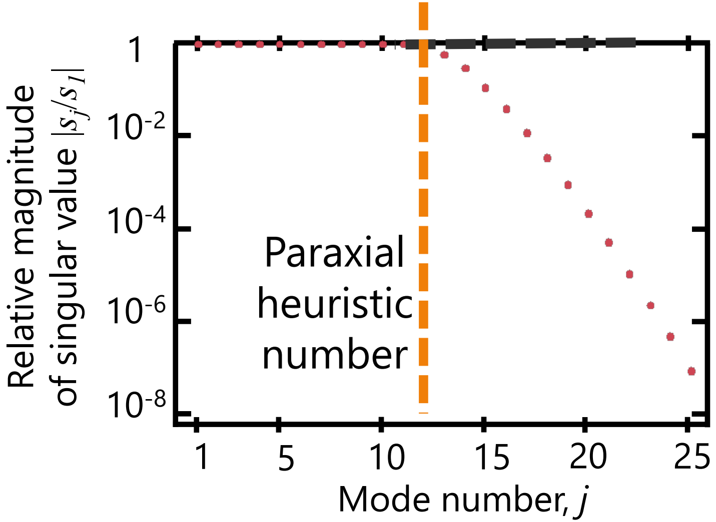
Mode 17
Relative magnitude of singular value
$|s_j/s_1|$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
15
20
25



Mode 19
Relative magnitude of singular value
$|s_j/s_1|$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10
15
20
25

Paraxial heuristic number and paraxial degeneracy





"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679 (2019)

Paraxial heuristic number

$N_H \sim W_S W_R / \lambda L$

for source and receiver widths W_S, W_R

separation L

wavelength λ

Once we pass the number we expect from conventional "diffraction limits"

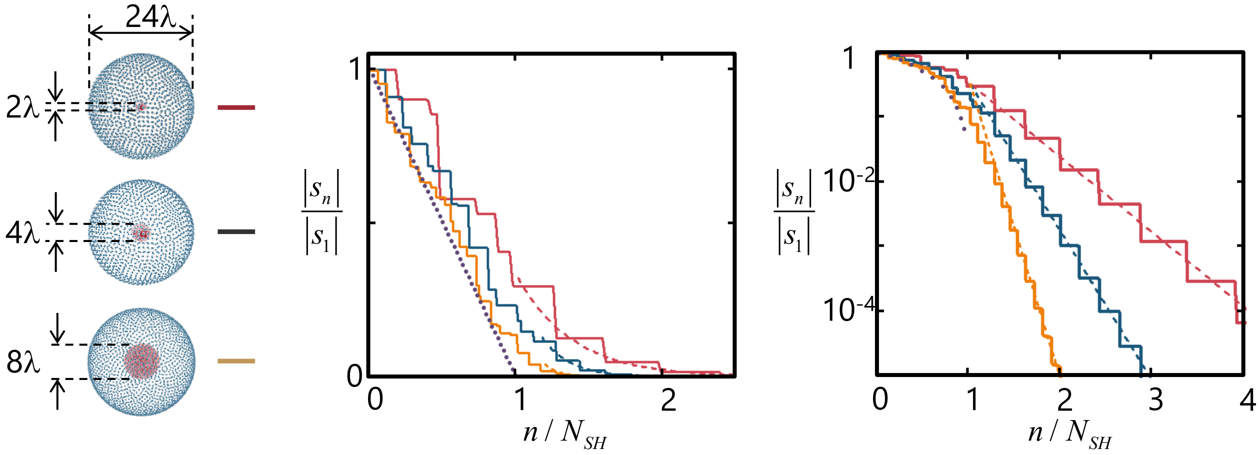
coupling strengths for further communication modes

fall off drastically and somewhat exponentially

We might think this is because the waves "miss" the receiving space

but that is not the general explanation

3D examples – concentric spherical shells



"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679 (2019)

Z. Kuang, D. A. B. Miller, and O. D. Miller, "Bounds on the Coupling Strengths of Communication Channels and Their Information Capacities," https://doi.org/10.48550/arXiv.2205.05150

Concentric spherical shell source and receiver spaces

are not easily analyzed by conventional "diffraction limit" theories

and do not show "paraxial degeneracy"

and the waves from the source space cannot "miss" the receiving space

but we still get some characteristic number of well-coupled communication modes

and a quasi-exponential fall-off of coupling beyond that

Why the abrupt fall-off past some number

Why do we *always* see

- regardless of the shape of the source and receiving volumes or surfaces
  - some number of "well coupled" channels
    - followed by an abrupt, quasi-exponential fall-off past this number
      - and just what gives this number?

We might argue this is just "diffraction"

- though that does not explain the concentric spheres case
  - where the waves cannot "miss" the receiving volume

Is there some underlying piece of physics we are missing?

Tunneling escape of waves

stanford.io/3X4Kk0Y

David Miller, *Stanford University*
Zeyu Kuang, Owen Miller, *Yale University*

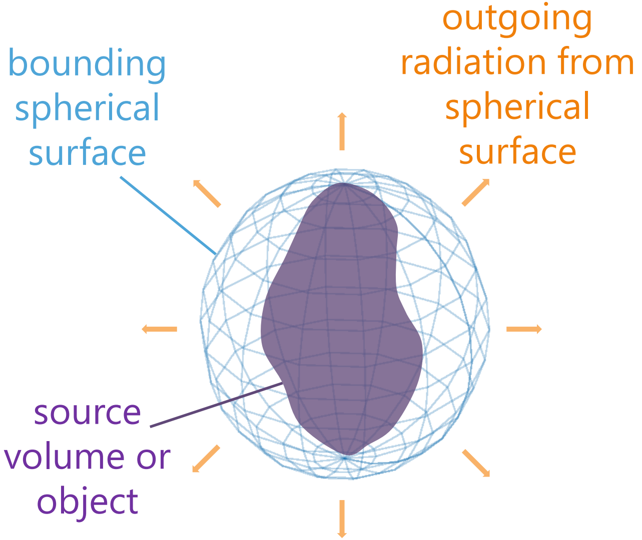
Waves from arbitrary volumes

How can we count the maximum number of well coupled waves (at a given frequency)

- from some finite volume?

Our approach

- Surround the volume with a mathematical "bounding" spherical surface
  - Count the number of well-coupled waves possible from this spherical surface
    - which then becomes the upper bound for waves from the source volume



D. A. B. Miller, Z. Kuang, O. D. Miller, "Tunneling escape of waves," http://arxiv.org/abs/2311.02744

Waves from arbitrary volumes

We show that, for spherical waves

with one key mathematical trick

there is a very simple and physical result

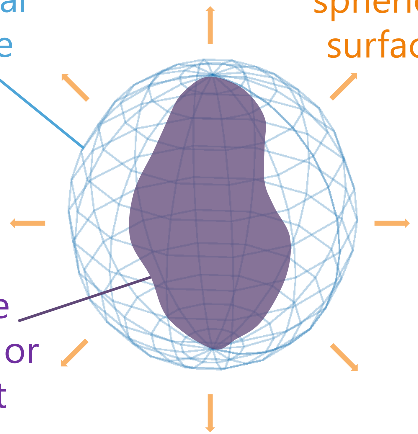
Beyond a certain simple threshold of "complexity" of spherical waves

they must "tunnel" to escape

Because the fall-off from tunneling is generally so rapid

this threshold effectively tells us the maximum number of well-coupled waves

and explains the quasi-exponential fall-off



D. A. B. Miller, Z. Kuang, O. D. Miller, "Tunneling escape of waves," http://arxiv.org/abs/2311.02744

Waves in spherical coordinates

In spherical coordinates r, θ, and ϕ

the solution to the wave equation separates to

$$U_{nm}(\mathbf{r}) = z_n(kr) Y_{nm}(\theta, \phi)$$

where $z_n(kr)$ is one of the spherical Bessel functions of order n, and

$Y_{nm}(\theta,\phi)$ is a spherical harmonic

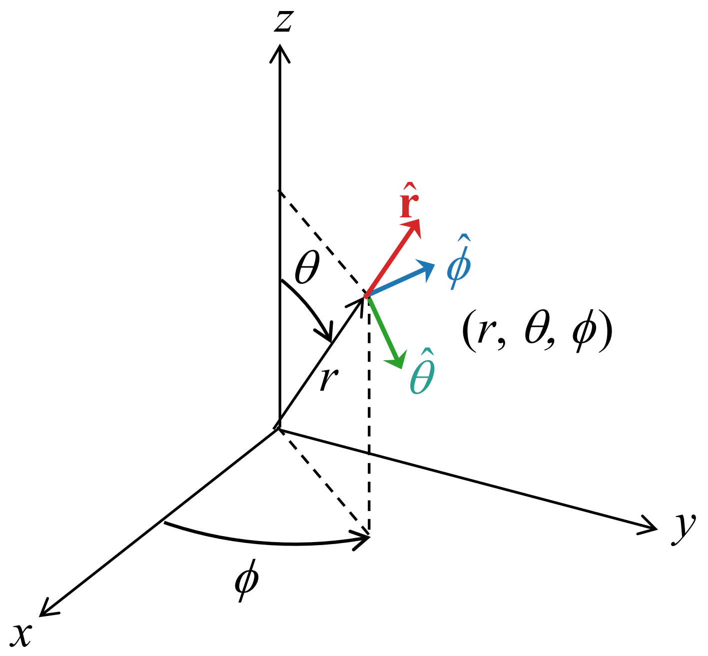
Here m and n are integers with

$n = 0,1,2,\ldots$ and $-n \le m \le n$

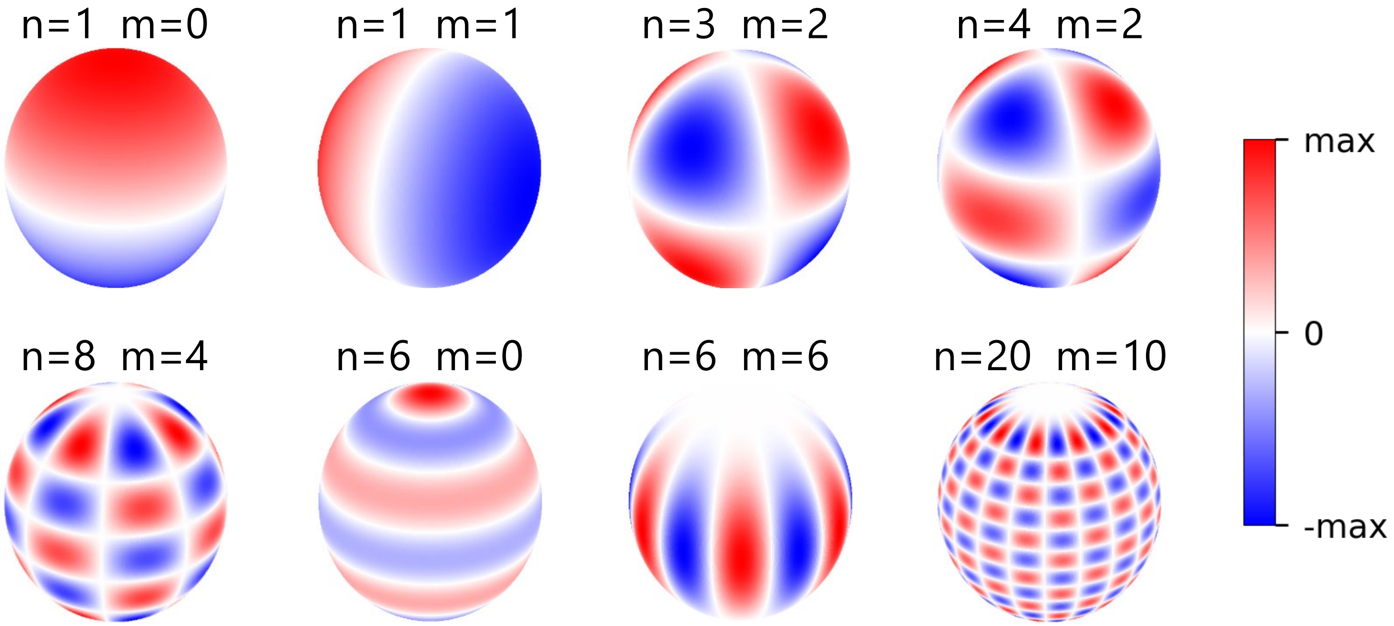
So, if we know the largest n for waves to propagate without tunneling

we can easily add up the total number of waves up to and including that n

$2n+1$ for each n



Spherical harmonics



Spherical harmonics are functions of angle only, and can be plotted on a spherical surface

They have *n* nodal circles altogether, with |*m*| through the poles (in their real form)

Escape radius

Specifically, for a given "order" n of spherical wave

there is an "escape radius"

$$r_{escn} = \frac{\sqrt{n(n+1)}}{k} \equiv \frac{\lambda_o}{2\pi}\sqrt{n(n+1)}$$

So, if the radius r_o of the spherical surface of interest

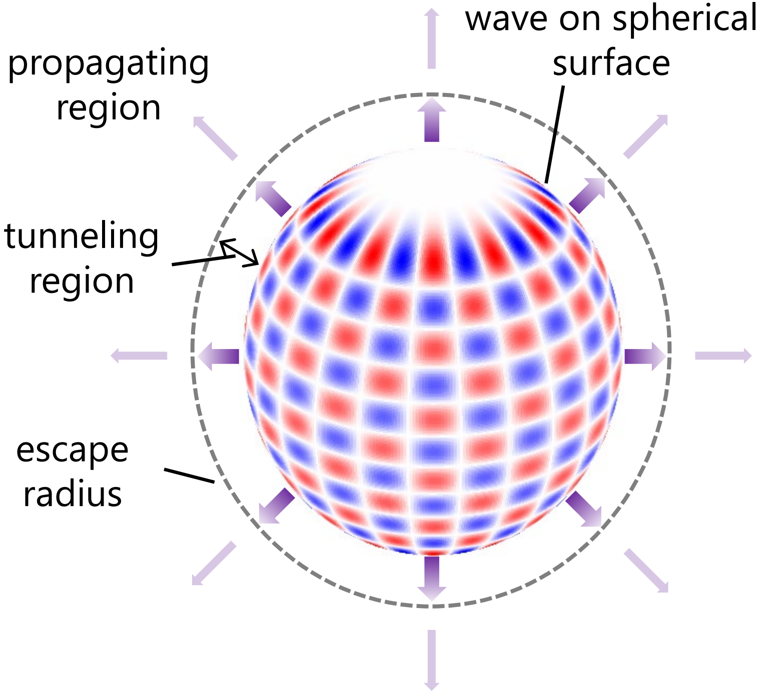
is smaller than the escape radius

for some order n of spherical wave

a wave with this n must tunnel

until it reaches the escape radius

after which it can propagate



D. A. B. Miller, Z. Kuang, O. D. Miller, "Tunneling escape of waves," http://arxiv.org/abs/2311.02744

Spherical Bessel functions and equation

Spherical Bessel functions obey

$$\rho^2 \frac{d^2 z_n(\rho)}{d\rho^2} + 2\rho \frac{dz_n(\rho)}{d\rho} + \left(\rho^2 - n(n+1)\right) z_n(\rho) = 0$$

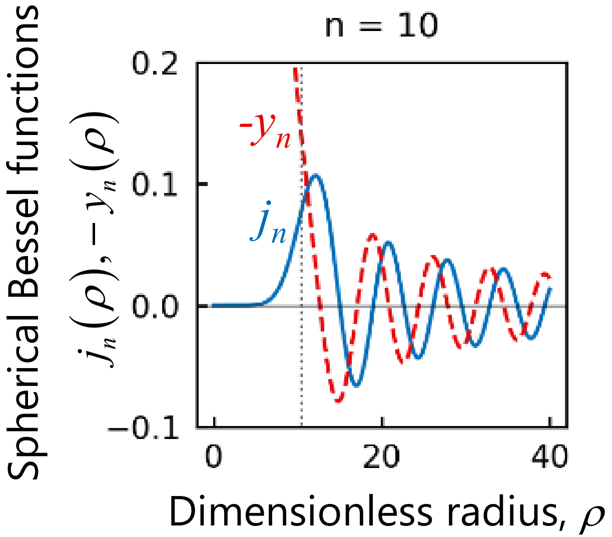
Classic radial standing wave solutions are

j_n which grows quasi-exponentially for small radii
and is quasi-oscillatory for larger radii

y_n which is singular at the origin
decaying quasi-exponentially for small radii
becoming quasi-oscillatory at large radii

Physically, ρ here is the dimensionless radius

$$\rho = kr \equiv 2\pi \frac{r}{\lambda}$$



Taking out the 1/radius dependence

Since the spherical Bessel functions have an underlying 1/radius dependence at large radius

- as appropriate for what are ultimately spherically expanding waves

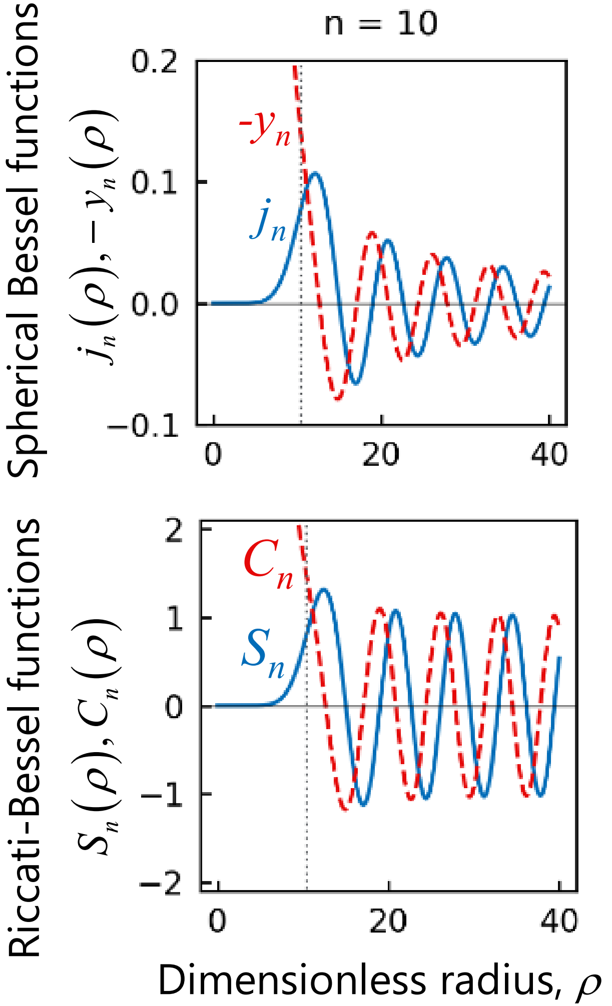
- it could be useful to remove that dependence
  - multiplying by radius
  - which gives functions corresponding to power per unit solid angle
    - rather than power per unit area

So, we recast in terms of such functions

- known as Riccati-Bessel functions

$$S_n(\rho) = \rho j_n(\rho) \qquad C_n(\rho) = -\rho y_n(\rho)$$

$$\xi_n(\rho) = \rho h_n^{(1)}(\rho) \equiv S_n(\rho) - iC_n(\rho)$$



Riccati-Bessel equation

Given that the spherical Bessel functions satisfy

$$\rho^2 \frac{d^2 z_n(\rho)}{d\rho^2} + 2\rho \frac{dz_n(\rho)}{d\rho} + \left(\rho^2 - n(n+1)\right) z_n(\rho) = 0$$

then we can easily check that all the Riccati-Bessel functions satisfy

$$\rho^2 \frac{d^2 \zeta_n}{d\rho^2} + \left(\rho^2 - n(n+1)\right)\zeta = 0$$

We can rearrange that to

$$-\frac{d^2 \zeta_n}{d\rho^2} + \frac{n(n+1)}{\rho^2}\zeta_n = \zeta_n$$

Riccati-Bessel "Schrödinger" equation

But wait!!!!!!

$$-\frac{d^2\zeta_n}{d\rho^2}+\frac{n(n+1)}{\rho^2}\zeta_n=\zeta_n$$

is in the form of a Schrödinger equation

$$-\frac{d^2\zeta_n}{d\rho^2}+V(\rho)\zeta_n=E_n\zeta_n$$

with effective radial potential

$$V(\rho)=\frac{n(n+1)}{\rho^2}$$

and the same "eigenenergy" $E_n=1$ for all n

Tunneling escape and escape radius

With the equation $$-\frac{d^2\zeta_n}{d\rho^2} + \frac{n(n+1)}{\rho^2}\zeta_n = \zeta_n$$

if the "potential energy" exceeds the "total energy", i.e., if

$$\frac{n(n+1)}{\rho^2} > 1$$ or, equivalently $$n(n+1) > \rho^2$$

the wave will be tunneling rather than propagating

So, for each n, there is an "escape radius" $$\rho_{escn} = \sqrt{n(n+1)}$$

or, equivalently, in dimensioned form $$r_{escn} = \frac{\sqrt{n(n+1)}}{k} \equiv \frac{\lambda_o}{2\pi}\sqrt{n(n+1)}$$

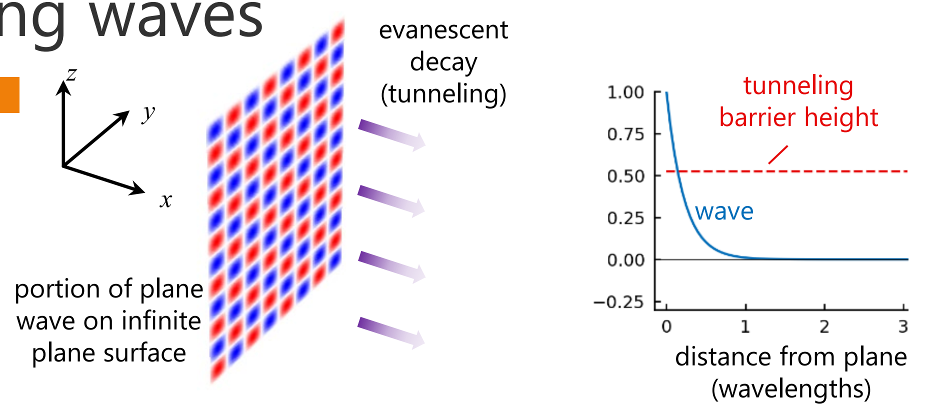
Evanescent and spherical escaping waves

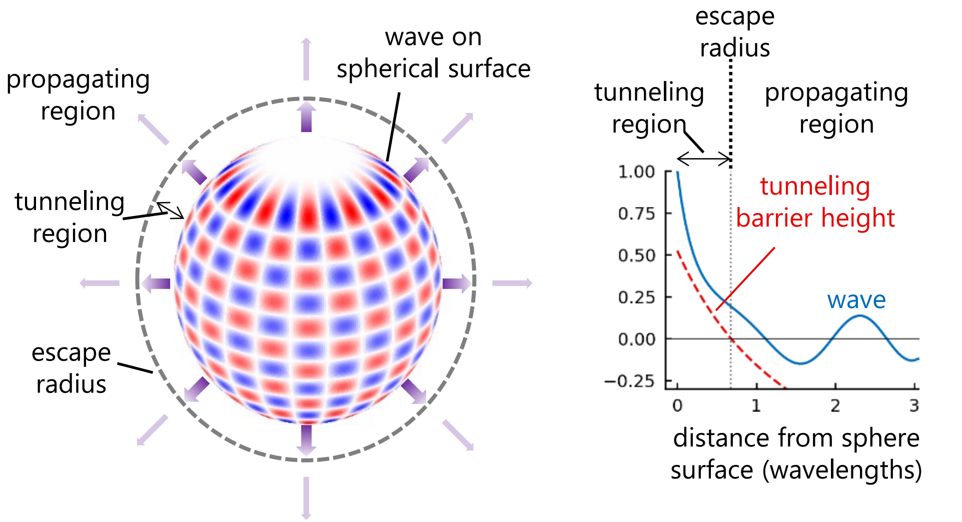
Both plane and spherical waves start with the same tunneling barrier height

- and hence the same initial decay
  - but the falling barrier height for the spherical wave
    - means it eventually escapes
      - to being a propagating wave

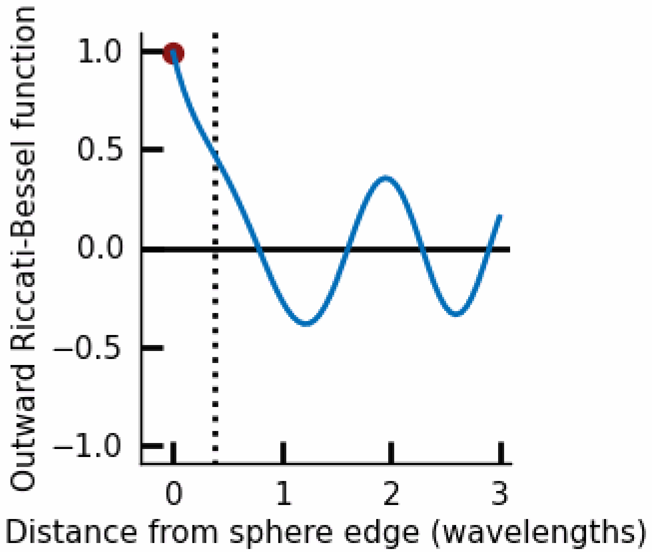
Note *all* such spherical waves

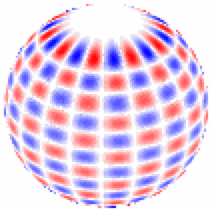
- eventually escape to some degree
  - though the evanescent plane wave
    - *never* does
      - **This is an artifact of the "infinite" extent of the plane wave**





Snapshot in time of a spherical wave





Real part of outward (Riccati- Bessel) spherical wave

Starting spherical surface radius 2.9 wavelengths

Wave with n = 20, m = 10, escape radius 3.26 wavelengths

Note the angular shape is constant as the wave expands

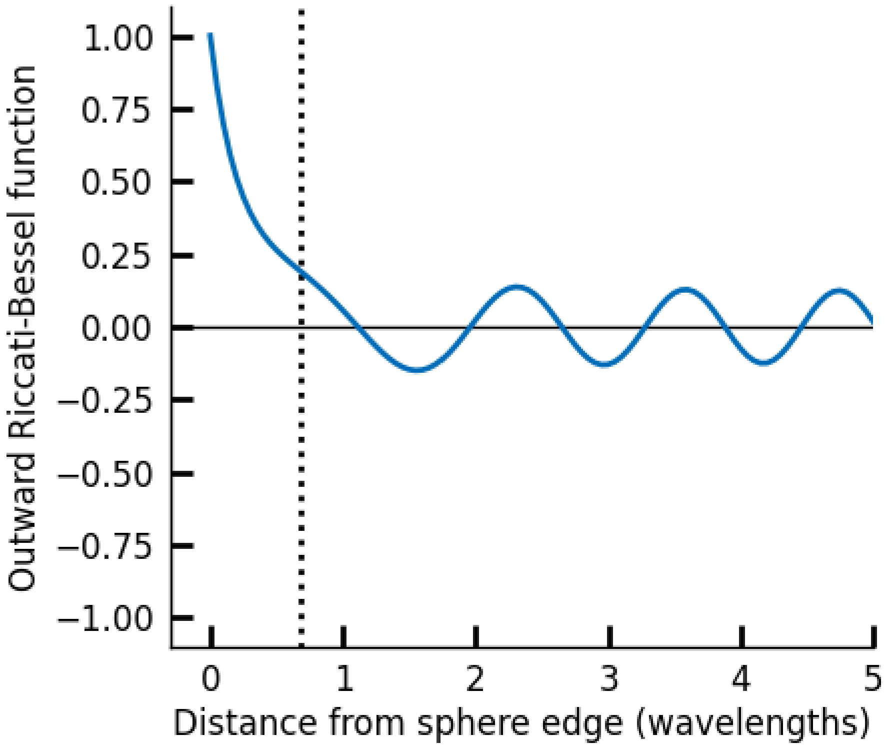
Outward wave propagation

As time progresses
- the wave beyond the escape radius
  - propagates outwards

We plot the outward Riccati-Bessel wave
- as a function of time
  - technically the real part of

$$\xi_n(2\pi r)\exp(-i\omega t)$$

  - normalized to unit amplitude at the sphere edge
- for a sphere of radius 2.9 wavelengths
  - with $n = 22$
    - which has an escape radius of 3.58 wavelengths



$r_o = 2.9 \quad n = 22 \quad r_{escn} = 3.58$

Outward wave propagation

As time progresses

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  - propagates outwards

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$$\xi_n(2\pi r)\exp(-i\omega t)$$

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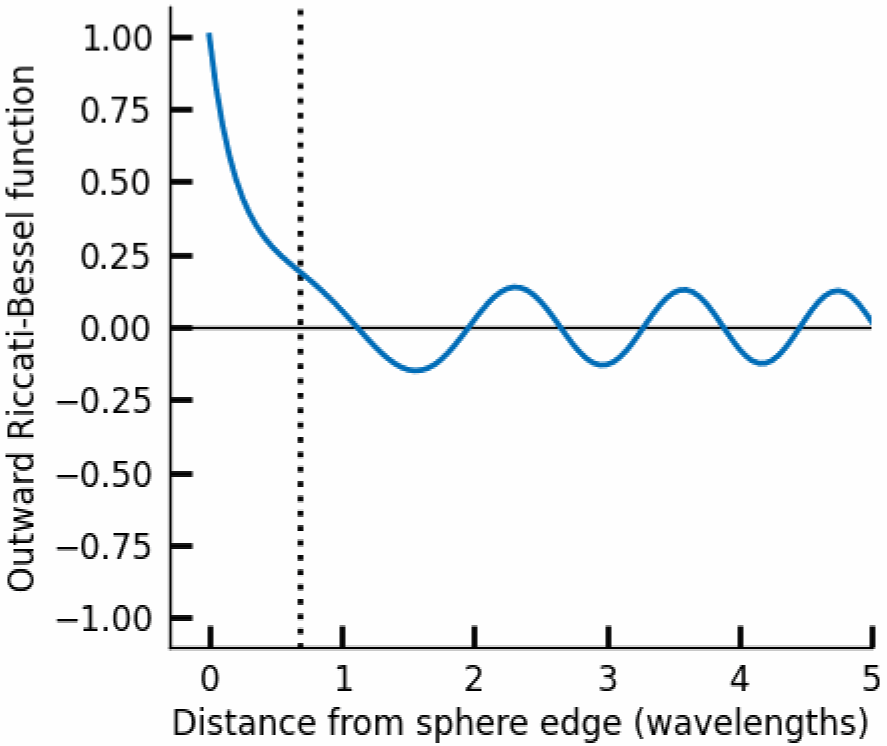


$$r_o = 2.9 \quad n = 22 \quad r_{escn} = 3.58$$

Spherical heuristic number

The threshold for tunneling is easy to characterize

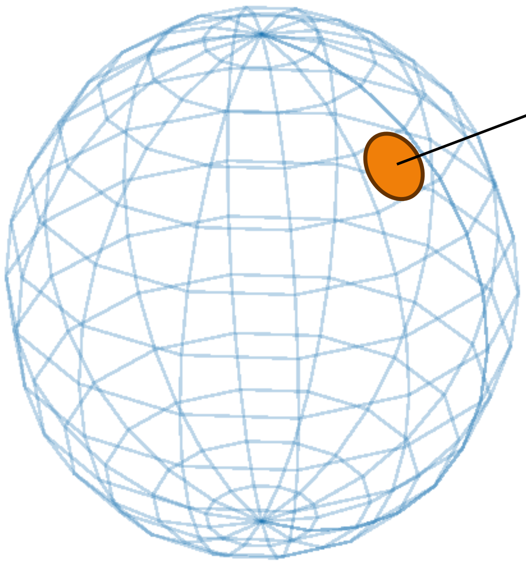
and gives a simple answer for the number of waves that do not need to tunnel

This is well approximated by the spherical heuristic number

$$N_{SH} = (kr_o)^2 \equiv \left(\frac{2\pi r_o}{\lambda_o}\right)^2 \equiv \frac{4\pi r_o^2}{(\lambda_o^2 / \pi)} \equiv \frac{A_S}{(\lambda_o^2 / \pi)}$$

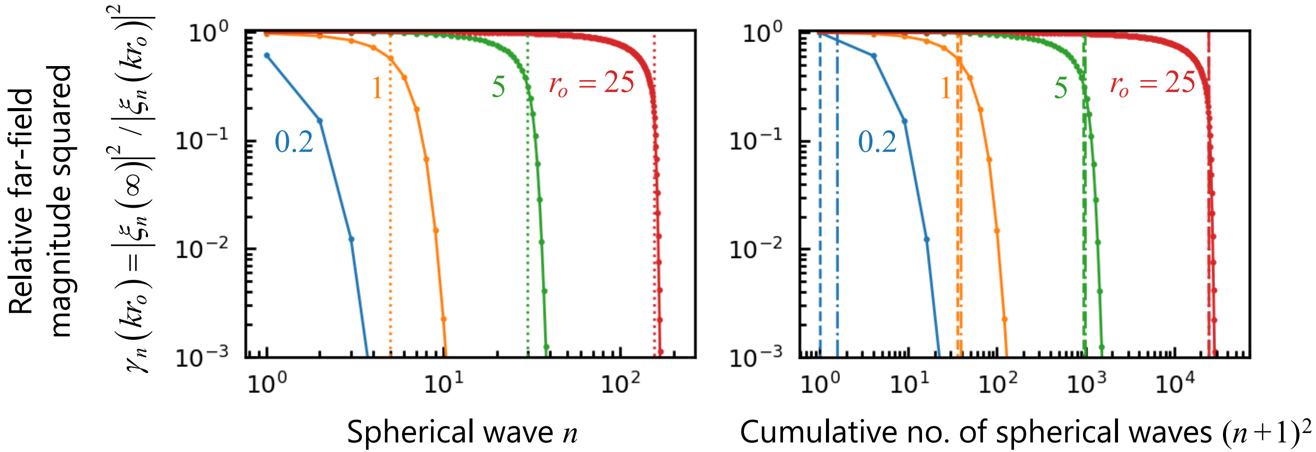
where A_S is the sphere area

so one "propagating" wave for every λ_o^2 / π of surface area



D. A. B. Miller, Z. Kuang, O. D. Miller, "Tunneling escape of waves," http://arxiv.org/abs/2311.02744

Relative far-field magnitude squared



dotted lines – n_p
dashed lines – N_{SH}
dot-dashed lines - N_p
(the exact counting result)

As the size of the spherical surface increases

the cut-off becomes increasingly relatively abrupt

tending towards the "absolutely abrupt" cut-off of evanescent waves

Note the spherical heuristic number N_{SH} is a good approximation to the total exact number N_p of "propagating" waves even down to ~ 1 wavelength of radius

Defining the diffraction limit

We can now construct a precise definition of

the "diffraction limit"

For a wave interacting with a volume

the wave passes the diffraction limit

if any spherical component of the wave must tunnel to enter or leave the bounding spherical surface enclosing the volume

Electromagnetic spherical outgoing waves

These have two transverse forms, separable in radial and angular parts

with the radial parts being the same as for the scalar case, so with

the same spherical/Riccati-Bessel tunneling and propagating behavior

and the angular part being a vector spherical harmonic function

$$\mathbf{C}_{mn}(\theta,\phi)=\nabla\times\left[\mathbf{r}Y_{nm}(\theta,\phi)\right]\equiv\nabla Y_{nm}(\theta,\phi)\times\mathbf{r}\qquad n=1,2\ldots\quad -n\le m\le n$$

giving "transverse electric" (TE) and "transverse magnetic" (TM) sets of waves

$$\mathbf{E}_{nm}^{(TE)}(r,\theta,\phi)=i\sqrt{\frac{\mu}{\varepsilon}}h_n^{(1)}(kr)\mathbf{C}_{mn}(\theta,\phi)\equiv i\sqrt{\frac{\mu}{\varepsilon}}\frac{\xi_n(kr)}{kr}\mathbf{C}_{mn}(\theta,\phi)$$

$$\mathbf{H}_{nm}^{(TM)}(r,\theta,\phi)=i\,h_n^{(1)}(kr)\mathbf{C}_{mn}(\theta,\phi)\equiv i\frac{\xi_n(kr)}{kr}\mathbf{C}_{mn}(\theta,\phi)$$

No n=0 electromagnetic outgoing waves

Note, because $\mathbf{C}_{mn}$ is a derivative of a spherical harmonic

and the spherical harmonic for $n = 0$ is uniform

there is no $n = 0$ wave in electromagnetism

If the first outgoing electromagnetic waves

(so, for $n = 1$)

are not to require tunneling to escape

the bounding spherical volume must be at least

$$r_{esc1} = \lambda_o / \left(\sqrt{2}\,\pi\right) \simeq 0.225\lambda_o$$

in radius or, equivalently, in diameter

$$d = \sqrt{2}\lambda_o / \pi \simeq 0.45\lambda_o$$

(consistent with the well-known Chu limit on antenna Q)

(Note: The escape radius for $n = 0$ acoustic waves is, however, zero

so, there is always one acoustic wave that can escape without tunneling

no matter how small the emitter or microphone)

Perfect cloaking - An optical “white hole”?

In this “white hole”, incoming light

- appears to be mostly “sucked” into the “white hole” in the middle
  - The phase fronts all “fall” rapidly into the “white hole”
- and then the light is regenerated
  - The phase fronts rapidly re-emerge from the “white hole”

How do we make this optical “white hole”?

- Note: it may be simpler than you think

Perfect cloacking - An optical “white hole”?

So, what does it take to build this cloak?

Absolutely nothing

- at least for this wave

If the wave is too complicated

- i.e., if it is trying to violate the “diffraction limit”
  - it can’t even effectively get into the volume
    - and it “reflects off free space”

This is the “inward wave” version of the tunneling escape

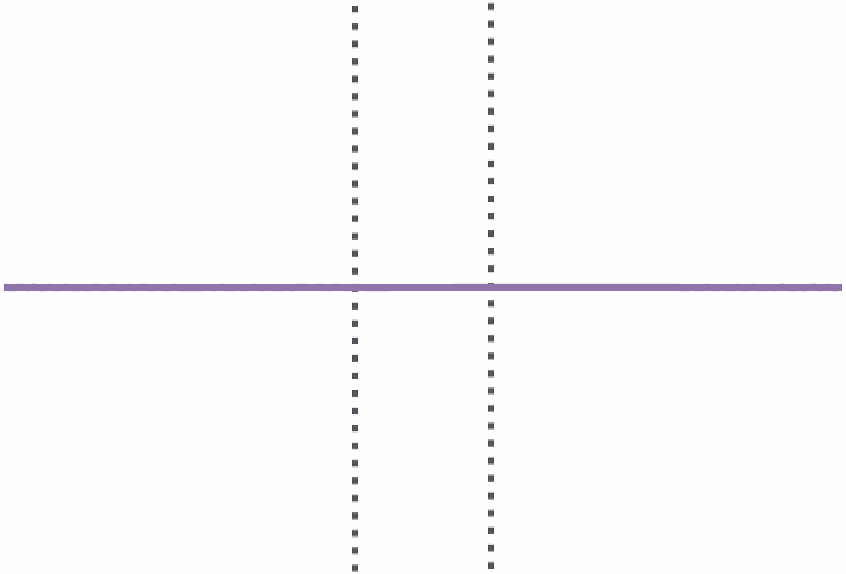
- with the wave trying to tunnel to get in

Interestingly

- the pulse actually looks as if it propagates right through!

Perfect cloaking?

Watch the blue dot, which propagates at the usual “phase velocity” of the wave



It appears to move right through the volume at a constant speed

Conclusions

New generations of programmable optics are emerging now

enabling a wide range of things we could not do before

and even setting themselves up

A new modal way of looking at optics

based on singular value decomposition

describes these and other optical devices

and gives us new fundamental and practical results and limits

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