Exploiting the linear algebra of optics

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Introduction

The linear algebra *of* optics and waves

- The right “modal” description of optics
  - “communication modes”
- lets us
  - analyze new kinds of complex and programmable optics
    - well beyond lenses, prisms and mirrors
  - understand optics and waves much more deeply
    - with fundamental and practical results and insights

Using optics to *perform* linear algebra, e.g., with interferometer meshes

- lets optics solve mathematical problems
- lets optics perform new physical functions on waves
  - and solve physical problems

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Modal optics

Ray optics

Imaging optics

Fourier optics

Resonator modes

Waveguide modes

Beams

Modal optics

Full electro-magnetic simulation

We want a "modal" optics

- to give the "right" most economical way to describe optical systems
  - The optimal sets of functions
    - with basic physical laws that apply only to them

To do this properly

- we need to move beyond "resonator" and "waveguide" modes
  - and even beyond standard "beams"

"Communicating with Waves Between Volumes … " Appl. Opt. **39**, 1681 (2000)

"Waves, modes, communications, and optics: a tutorial," Adv. Opt. Photon. **11**, 679-825 (2019)

A different way of thinking about modes and waves

When we look generally at

communications with waves

or scatterers, optical devices, or nanostructures

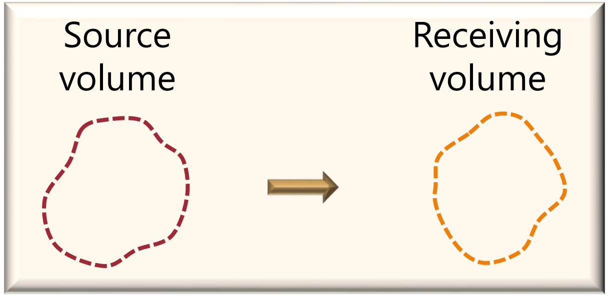
we need a different kind of "mode" that looks at

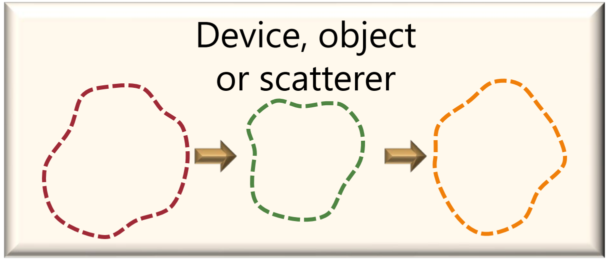
- "source" or input spaces
- and "receiving" or output spaces

They are "modes" in **two** spaces

not one space

They are **not** the "beams" between the spaces





"Waves, modes, communications, and optics: a tutorial," Adv. Opt. Photon. **11**, 679-825 (2019)

The rigorous approach to channels between volumes

Given some coupling operator or Green's function G_{SR} between sources and receivers

we want the orthogonal source functions $|\psi_{Sj}\rangle$

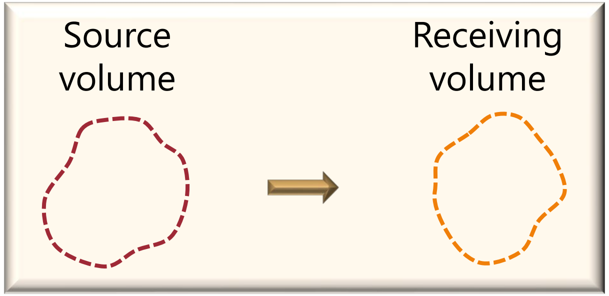
that couple, one by one, to orthogonal received waves $|\phi_{Rj}\rangle$

with some coupling strength s_j

These pairs of functions $|\psi_{Sj}\rangle$ and $|\phi_{Rj}\rangle$

are the "communication modes", which we find by

singular value decomposition (SVD) of G_{SR}



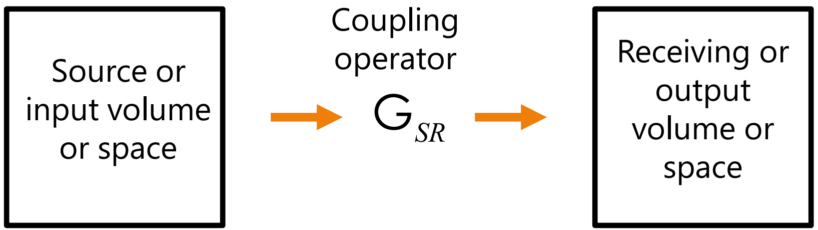
"Waves, modes, communications and optics," Adv. Opt. Photon. 11, 679-825 (2019)

Source or input volume V_S or space H_S → V_S, $|\psi_S\rangle$, H_S → G_{SR} → V_R, $|\phi_R\rangle$, H_R → Receiving or output volume V_R or space H_R

Modal optics

These communication modes
- completely and uniquely define
  - all the orthogonal channels in the system
    - e.g., for communication or sensing
    - e.g., for understanding limits to numbers and strengths of channels and couplings

There are no better orthogonal channels
- If we can’t do something using these channels
  - then we can’t do it any other way with the same optics



"Waves, modes, communications, and optics," Adv. Opt. Photon. **11**, 679 (2019)

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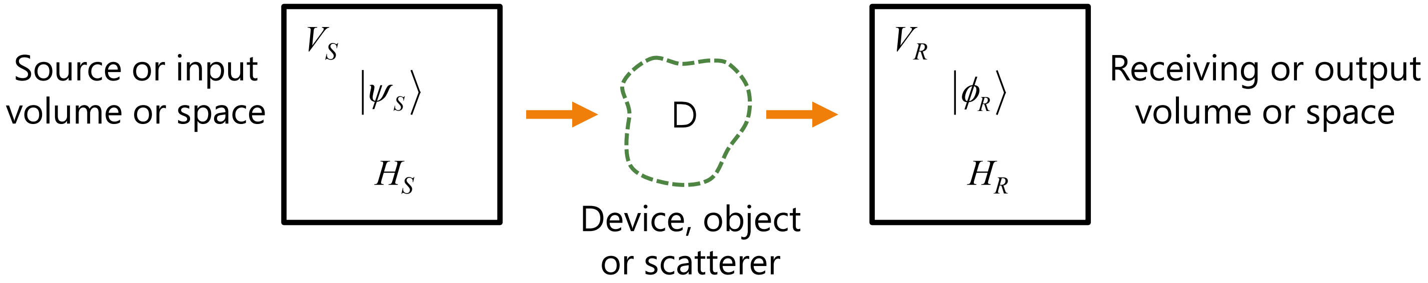
Mode-converter basis sets

"All linear optical devices are mode converters," Opt. Express **20**, 23985 (2012)



For any given linear optical device

described by a linear operator D

because we can perform the SVD of any linear operator D

we have what we can call

the **mode-converter basis sets** of functions

a set of orthogonal source functions that lead, one by one

to a set of corresponding orthogonal received waves

These fully describe the optical device as it operates between these spaces

Note this means there is a set of orthogonal channels through any linear optics

Results from this modal optics

This universal "modal" way of looking at optics

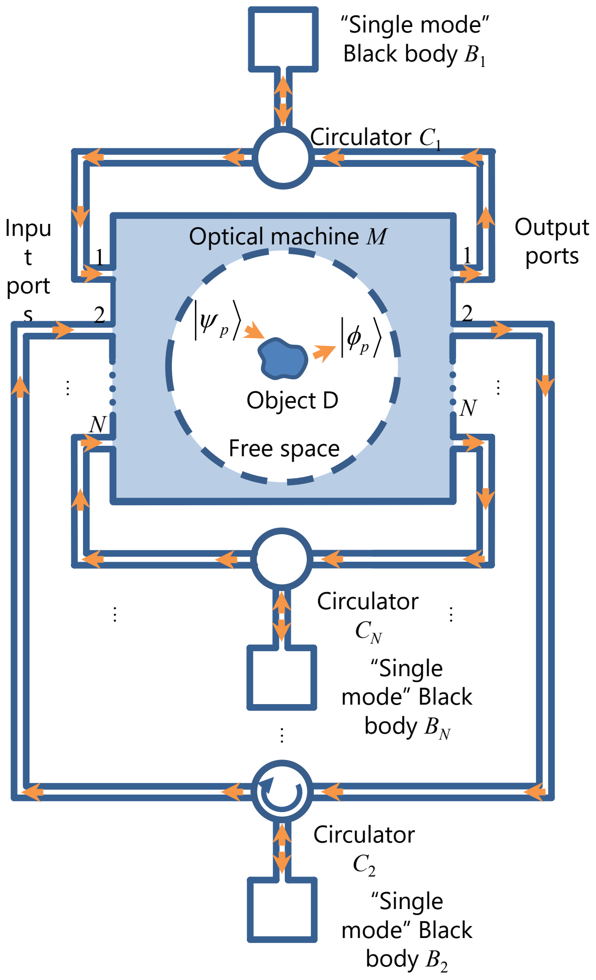
also allows basic wave results

- the correct (and modal) Kirchhoff radiation laws for thermal emission

A "though experiment" machine that leads to modal radiation laws for arbitrary objects, including non-reciprocal ones.

From the resulting modal radiation laws, we can deduce new radiation laws, including the ones we thought we had but actually didn't.

"Universal modal radiation laws for all thermal emitters," PNAS **114**, 4336 (2017)



Results from this modal optics

This universal "modal" way of looking at optics

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- the correct Kirchhoff radiation laws for thermal emission
- a simpler "modal" version of Einstein's A&B coefficient argument

"Waves, modes, communications, and optics," Adv. Opt. Photon. **11**, 679 (2019)

Results from this modal optics

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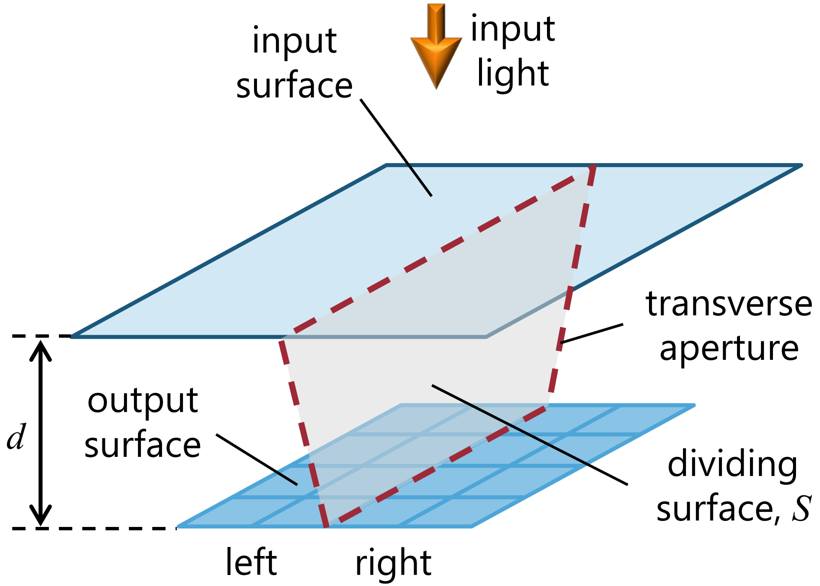
- the correct Kirchhoff radiation laws for thermal emission
- a simpler "modal" version of Einstein's A&B coefficient argument
- why optics needs thickness



The "transverse aperture" has to be large enough to support the communication modes from the left (right) input surface to the right (left) output surface, leading to bounds on thickness

"Why optics needs thickness," Science **379**, 41-45 (2023)

Results from this modal optics

This universal "modal" way of looking at optics

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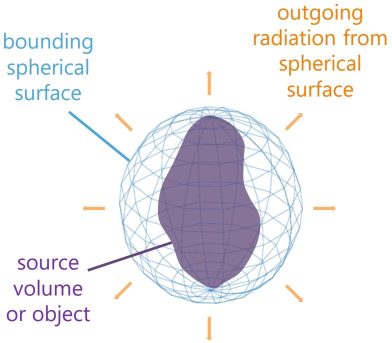
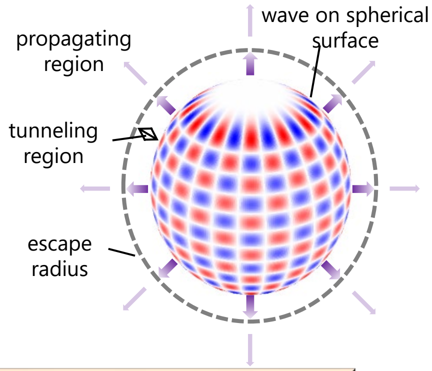
- the correct Kirchhoff radiation laws for thermal emission
- a simpler "modal" version of Einstein's A&B coefficient argument
- why optics needs thickness
- the real reason for diffraction limits

    tunneling escape of waves





Waves on the spherical bounding surface that are too complicated have to tunnel to escape

Waves that are too complicated would have to tunnel to get into small volumes and are instead effectively reflected by free space

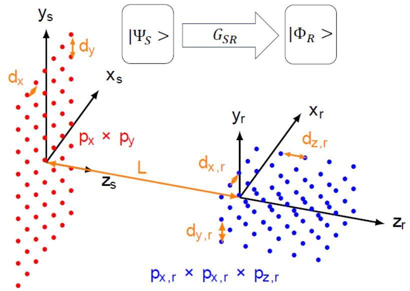
"Tunneling escape of waves," Nat. Photon. **19**, 284–290 (2025)

Results from this modal optics

This universal "modal" way of looking at optics

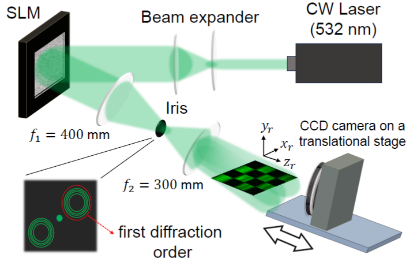
also allows basic wave results

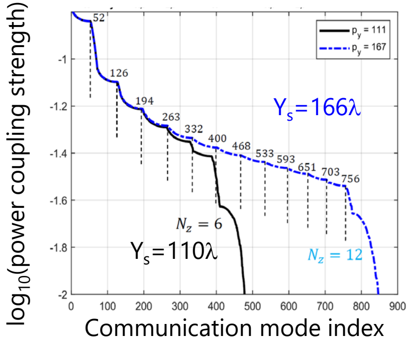
- the correct Kirchhoff radiation laws for thermal emission
- a simpler "modal" version of Einstein's A&B coefficient argument
- why optics needs thickness
- the real reason for diffraction limits
  - tunneling escape of waves
- how to form fields in volumes

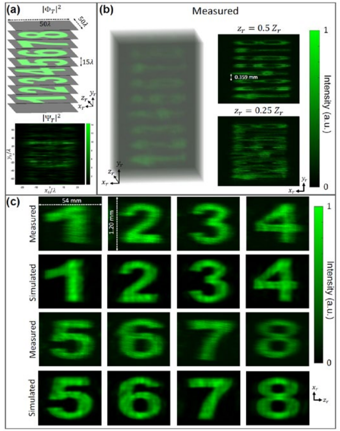


rectangular optical source area

cuboidal optical receiving volume







Communication modes let us calculate exactly what sources are required to generate a desired volume field, and tell us if it can't practically be generated

V. S. de Angelis et al., "Optimal structured light waves generation in 3D volumes using communication mode optics" arXiv:2411.10865

Results from this modal optics

This universal "modal" way of looking at optics

also allows basic wave results

- the correct Kirchhoff radiation laws for thermal emission
- a simpler "modal" version of Einstein's A&B coefficient argument
- why optics needs thickness
- the real reason for diffraction limits

  tunneling escape of waves
- how to form fields in volumes

and is also being used increasingly by the wireless community as it tries to understand spatial channels for 5G and 6G

Programmable photonics enables us controllably to operate on and with these "correct" modes in optics

"Universal modal radiation laws for all thermal emitters," PNAS **114**, 4336 (2017)

"Waves, modes, communications, and optics," Adv. Opt. Photon. **11**, 679 (2019)

"Why optics needs thickness," Science **379**, 41-45 (2023)

"Tunneling escape of waves," Nat. Photon. **19**, 284–290 (2025)

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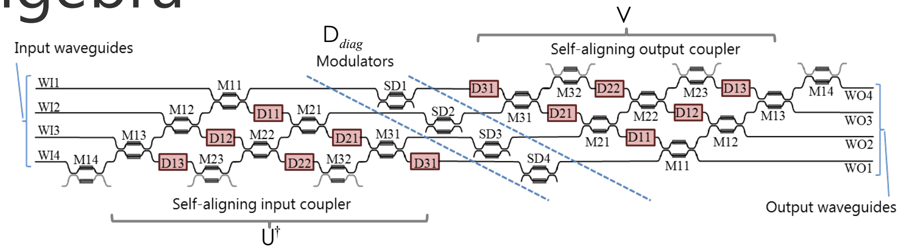
Using optics to *perform* linear algebra



Using optics to *perform* linear algebra, e.g., with interferometer meshes to solve mathematical problems

- arbitrary matrix multiplication
  - SVD architecture

"All linear optical devices are mode converters," Opt. Express **20**, 23985 (2012)

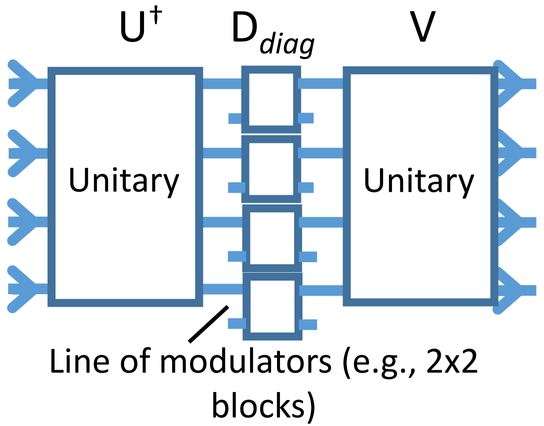
"Self-configuring universal linear optical component," Photon. Res. **1**, 1-15 (2013)

Any matrix or linear operator D can be rewritten using its singular value decomposition (SVD)

$$D = V D_{diag} U^{\dagger} \text{ or } D = \sum_m s_m |\phi_m\rangle\langle\psi_m|$$

U and V are unitary operators

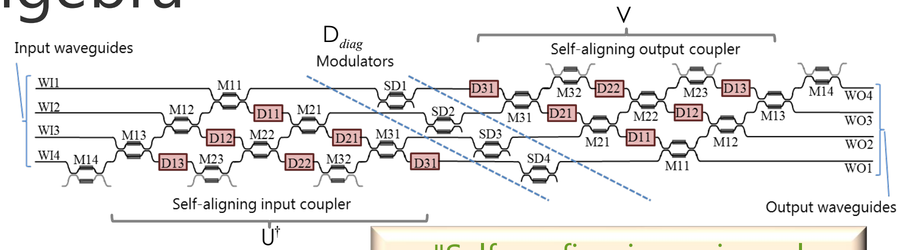
D_{diag} is a diagonal matrix of "singular values" s_m



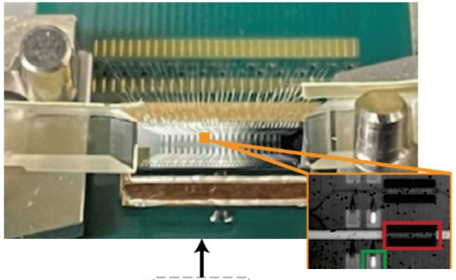
Using optics to *perform* linear algebra

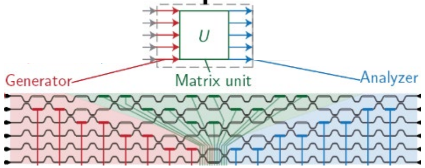
Using optics to *perform* linear algebra, e.g., with interferometer meshes

- have optics solve mathematical problems
  - arbitrary matrix multiplication
    - SVD architecture
    - generator-matrix-analyzer architecture



"Self-configuring universal linear optical component," Photon. Res. **1**, 1-15 (2013)

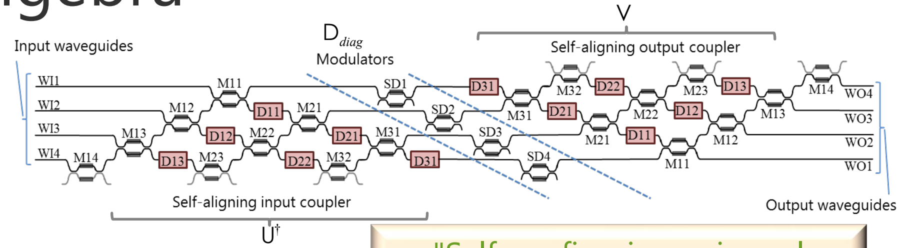




Using optics to *perform* linear algebra

Using optics to *perform* linear algebra, e.g., with interferometer meshes

- have optics solve mathematical problems
  - arbitrary matrix multiplication
    - SVD architecture
    - generator-matrix-analyzer architecture
  - neural network processors



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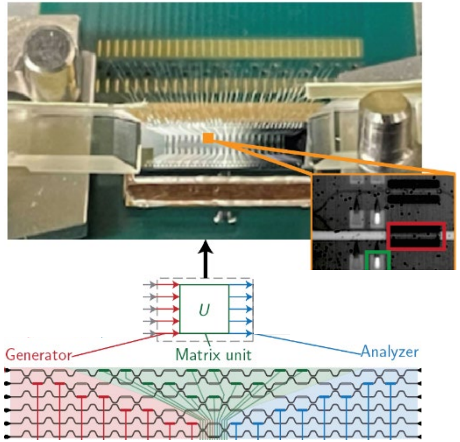
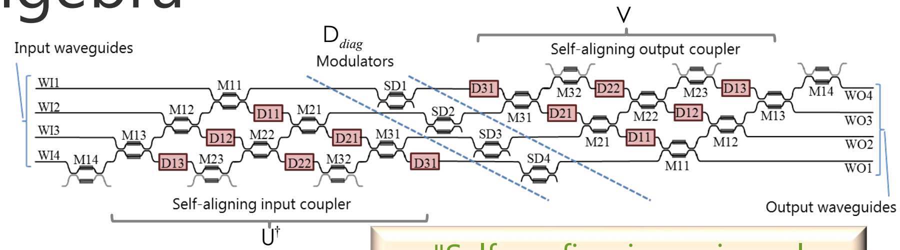


S. Pai et al., "Experimentally realized in situ backpropagation for deep learning in photonic neural networks," Science **380**, 398-404 (2023)

Using optics to *perform* linear algebra

- Using optics to *perform* linear algebra, e.g., with interferometer meshes
  - have optics solve mathematical problems
  - arbitrary matrix multiplication
    - SVD architecture
    - generator-matrix-analyzer architecture
  - neural network processors
  - inverting matrices



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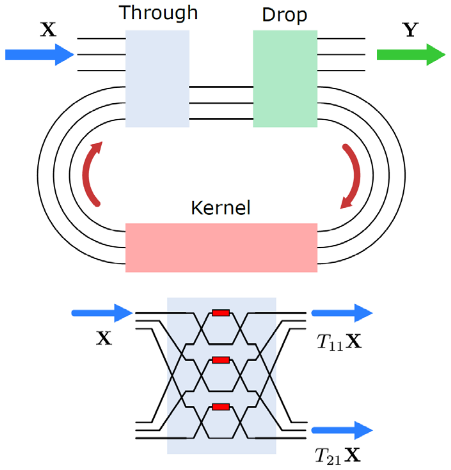
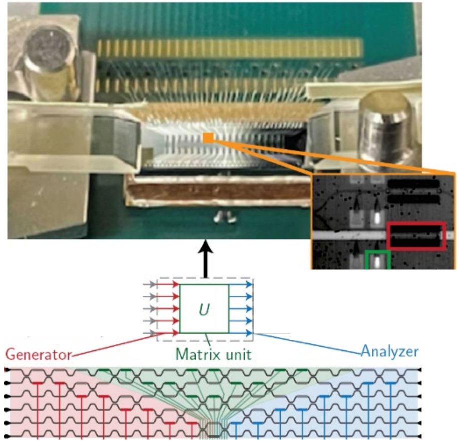


G. Cavicchioli et al., "Programmable integrated photonic circuit for matrix inversion," OFC 2024, Th1A.2.



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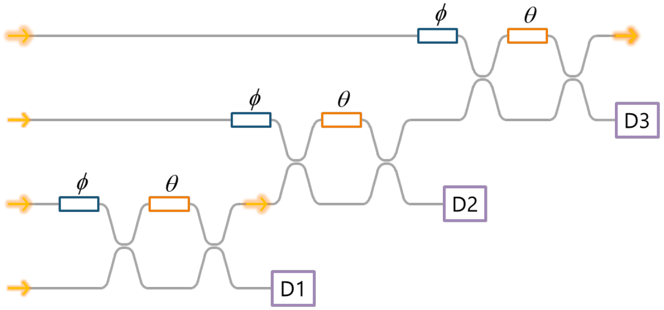
Using optics to *perform* linear algebra

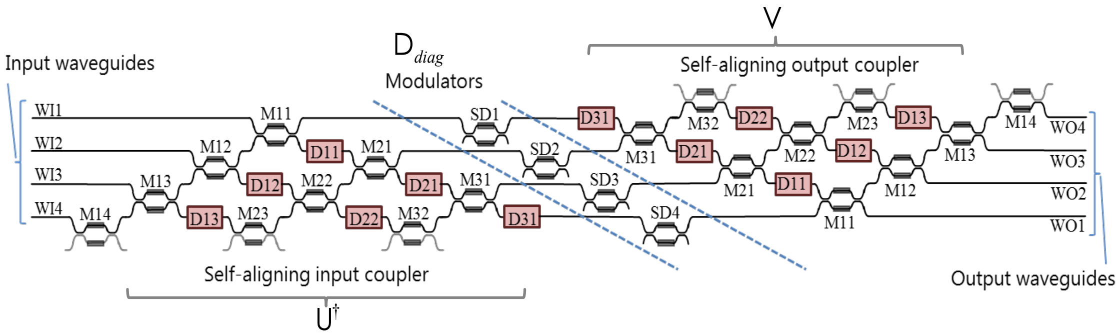
have optics perform new physical functions and operations on waves

- automatic beam combination and self-alignment
- arbitrary linear optical components



"Self-aligning universal beam coupler," Opt. Express **21**, 6360 (2013)

This "self-configuring layer" of Mach-Zehnder interferometers can automatically combine the input powers to the output just by successive power minimization on the detectors D1 – D3



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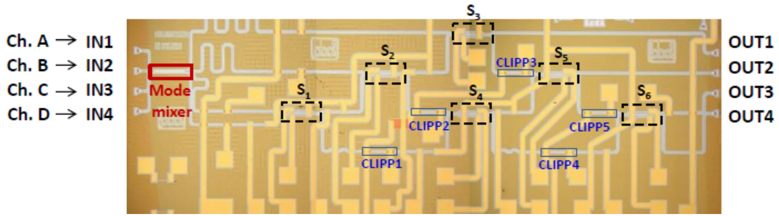
Using optics to *perform* linear algebra

have optics perform new physical functions and operations on waves

automatic beam combination and self-alignment

arbitrary linear optical components

automatically separating overlapping modes



A. Annoni et al., "Unscrambling light – automatically undoing strong mixing between modes," Light Science & Applications 6, e17110 (2017)

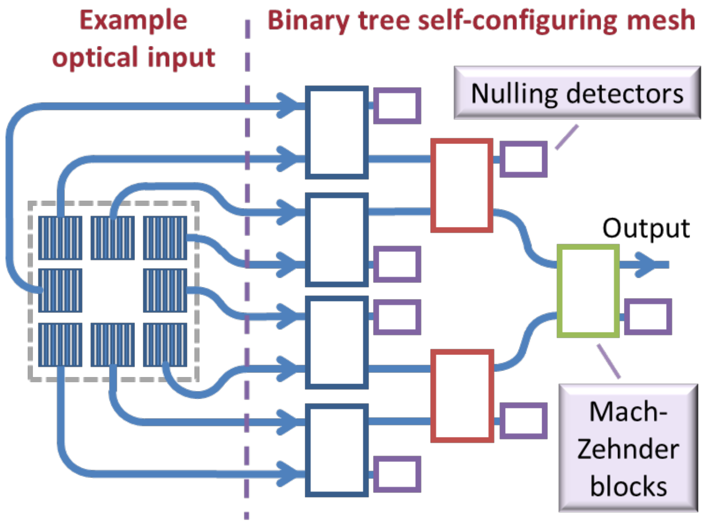
See also A. Ribeiro et al., "Demonstration of a 4 × 4-port universal linear circuit," Optica 3, 1348 (2016)

This circuit, a succession of self-configuring layers forming a "triangular" mesh, can automatically separate out overlapping, mixed optical signals to its outputs

Using optics to *perform* linear algebra

have optics perform new physical functions and operations on waves

- automatic beam combination and self-alignment
- arbitrary linear optical components
- automatically separating overlapping modes
- generation of arbitrary beams
- reference-free measurement of arbitrary beams



Self-configuring this "binary tree" layer to route all power to the output automatically measures the relative amplitudes and phases of the input light, with the values deduced from the resulting mesh settings.

Run backwards, it can generate any beam emerging from the "inputs".

"Analyzing and generating multimode optical fields using self-configuring networks," Optica 7, 794 (2020)

See also J. Bütow et al. "Spatially resolving amplitude and phase of light with a reconfigurable photonic integrated circuit," Optica 9, 939 (2022)

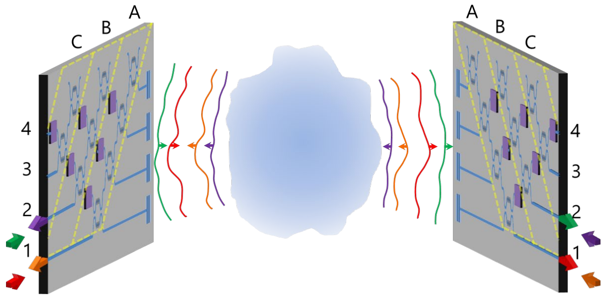
Using optics to *perform* linear algebra

have optics perform new physical functions and operations on waves

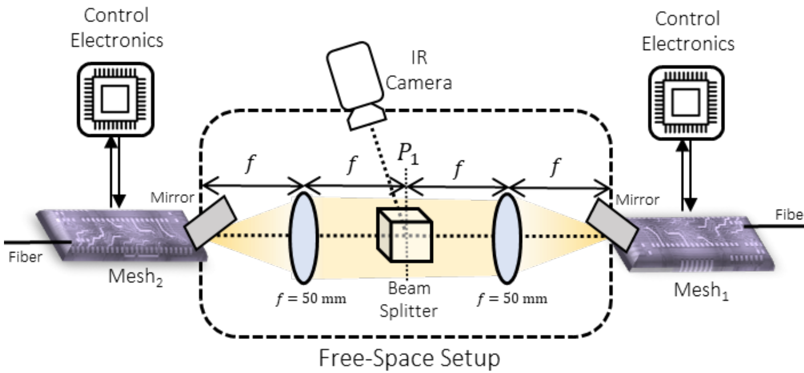
- automatic beam combination and self-alignment
- arbitrary linear optical components
- automatically separating overlapping modes
- generation of arbitrary beams
- reference-free measurement of arbitrary beams
- automatically finding the best orthogonal channels through any optics

By power maximizing on rows of the mesh at both sides, this circuit can automatically find the best orthogonal channels between the two sides, physically performing the singular-value decomposition of the optical system

This is a true optical computer!



"Establishing optimal wave communication channels automatically," J. Lightwave Technol. 31, 3987 (2013)



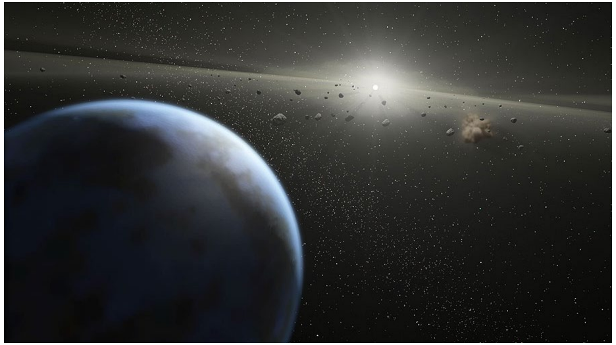
S. SeyedinNavadeh et al., "Determining the optimal communication channels of arbitrary optical systems using integrated photonic processors," Nat. Photon. **18**, 149-155 (2024)

Using optics to *perform* linear algebra

have optics perform new physical functions and operations on waves

- automatic beam combination and self-alignment
- arbitrary linear optical components
- automatically separating overlapping modes
- generation of arbitrary beams
- reference-free measurement of arbitrary beams
- automatically finding the best orthogonal channels through any optics
- help detect exoplanets by optimized modal filtering

Dan Sirbu et al., "AstroPIC: near-infrared photonic integrated circuit coronagraph architecture for the Habitable Worlds Observatory," Proc. SPIE 13092, 130921T (2024)

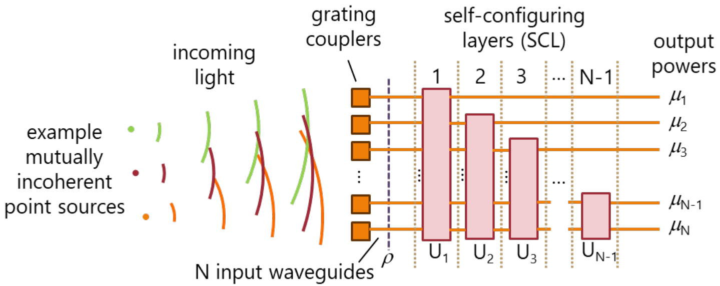


Use a programmable photonic mesh to provide optimal modal filtering to reject star light and pass possible exoplanet light

Using optics to *perform* linear algebra

Roques-Carmes et al., "Measuring, processing, and generating partially coherent light ..." LSA **13**, 260 (2024)

- have optics perform new physical functions and operations on waves
  - automatic beam combination and self-alignment
  - arbitrary linear optical components
  - automatically separating overlapping modes
  - generation of arbitrary beams
  - reference-free measurement of arbitrary beams
  - automatically finding the best orthogonal channels through any optics
  - help detect exoplanets by optimized modal filtering
  - measuring and separating partially coherent light into its mutually incoherent and orthogonal components



With partially coherent input light
- by power maximizing on the successive self-configuring layers
  - this circuit can measure the coherency matrix of that light
    - simultaneously separating it into its mutually incoherent and mutually orthogonal components

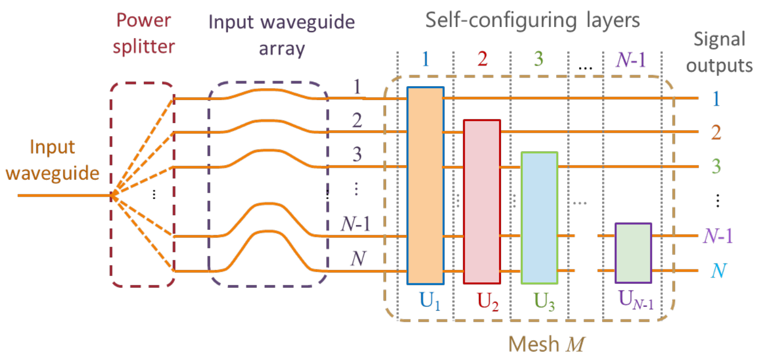
No other known apparatus can apparently perform this separation

Using optics to *perform* linear algebra

"Universal programmable and self-configuring optical filter"
arXiv.2501.11811

have optics perform new physical functions and operations on waves

- automatic beam combination and self-alignment
- arbitrary linear optical components
- automatically separating overlapping modes
- generation of arbitrary beams
- reference-free measurement of arbitrary beams
- automatically finding the best orthogonal channels through any optics
- help detect exoplanets by optimized modal filtering
- measuring and separating partially coherent light into its mutually incoherent and orthogonal components
- arbitrarily programmable and self-configuring spectrometers



This circuit can function like an arrayed waveguide grating filter, but can also

- implement any linear combination of such filter functions
- self-configure to specific wavelengths
- reject N-1 arbitrary wavelengths

Conclusions

The correct modal linear optics view is allowing us to

- prove many new basic insights about optics
  - opening new applications
    - including especially interferometer mesh architectures
      - which allow mathematical calculations in optics
        - and many new physical optical systems and uses

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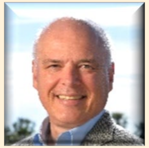
Major collaborators

- Groups of Andrea Melloni, Francesco Morichetti, and Marco Sampietro, Politecnico di Milano
- Groups of Olav Solgaard and Shanhui Fan, Stanford
- Owen Miller and Zeyu Kuang, Yale
- Capasso group, Harvard
- Nader Engheta, U. Penn.
- Dan Sirbu and Rus Belikov, NASA

Funding from AFOSR



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