Oscillations and waves 1

Classical mechanics

Modern physics for engineers

David Miller

Introduction

Before moving on

we remind ourselves of basic classical mechanics, such as

- kinetic energy
- momentum
- Newton’s second law
- potential energy and force

and various relations between these

Introduction

- Then we will look at simple oscillators
  - such as a mass on a spring
- Finally, we will look at
  - waves on a string
    - including ideas like standing waves
- We will also introduce the important idea of "modes"

Momentum and kinetic energy

Momentum and kinetic energy

For a particle of mass m

the classical momentum

which is a vector

because it has direction

is $\mathbf{p} = m\mathbf{v}$

where $\mathbf{v}$ is the (vector) velocity

The kinetic energy

the energy associated with motion

is

$$K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Momentum and kinetic energy

In the kinetic energy expression

$$K.E. = \frac{p^2}{2m}$$

we mean

$$p^2 \equiv \mathbf{p} \cdot \mathbf{p}$$

i.e.,

the vector dot product of $\mathbf{p}$ with itself

Potential energy

Potential energy

Potential energy is defined as

energy due to position

It is usually denoted by V in quantum mechanics

even though this potential energy

in units of Joules

might be confused with the idea of voltage

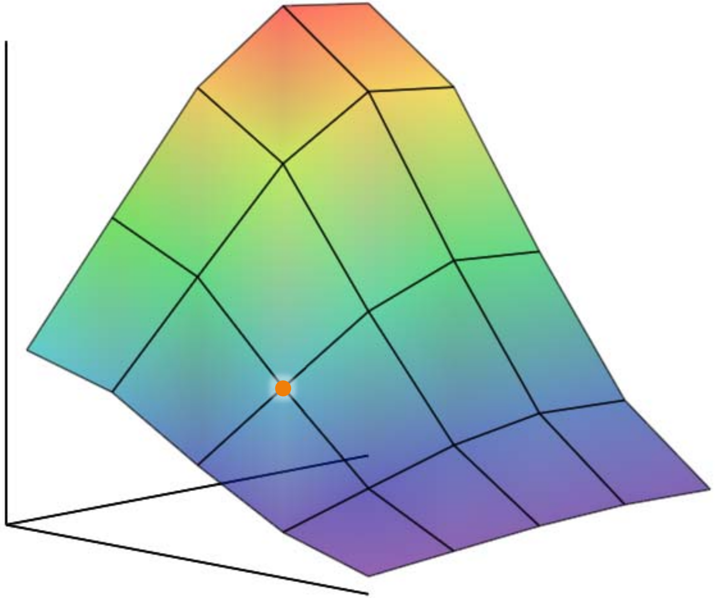
in units of Joules/Coulomb

and even though we use voltage often in quantum mechanics

Potential energy

Since it is energy due to position

- it can be written as $V(\mathbf{r})$
  - We can talk about potential energy
    - if that energy only depends on where we are
      - not how we got there



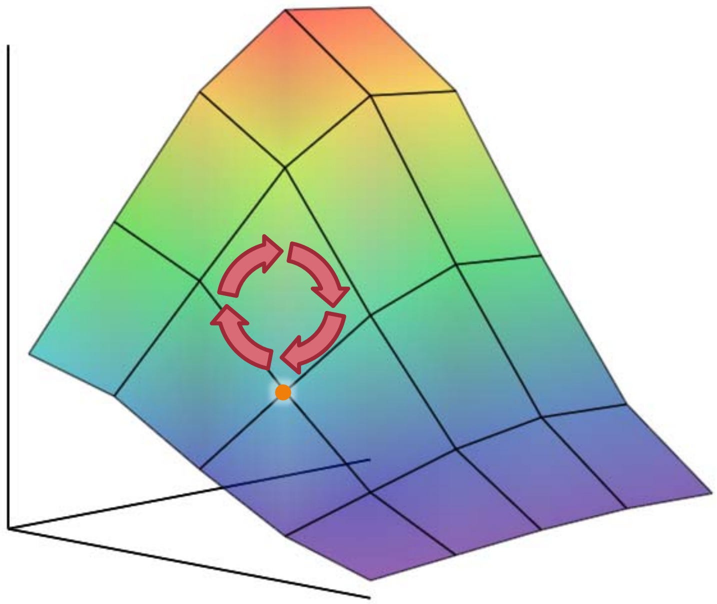
Potential energy

Classical "fields" with this property are called

- "conservative" or "irrotational"
  - the change in potential energy round any closed path is zero

Not all fields are conservative

- e.g., going round a vortex
- but many are conservative
  - gravitational, electrostatic



Energy origin

- The “zero” or “origin” we use for potential energy is always arbitrary
  - We can choose it to be what we want
    - as long as we are consistent
      - There is no absolute origin for potential energy

Energy origin

We only really work with differences in potential energy

between one position and another

and we can choose any “zero” position we want

as long as we are consistent

Force and potential energy

Force

In classical mechanics

we often use the concept of force

Newton's second law relates force and acceleration

$$\mathbf{F} = m\mathbf{a}$$

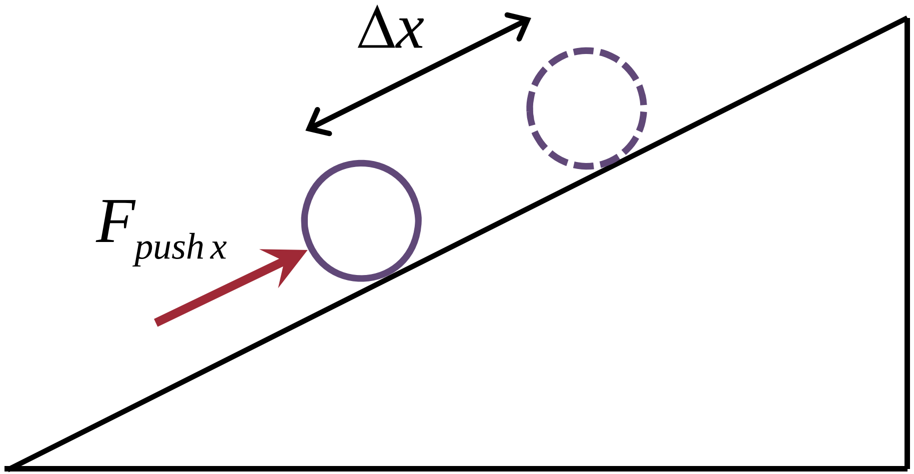
where m is the mass and $\mathbf{a}$ is the acceleration

Equivalently

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

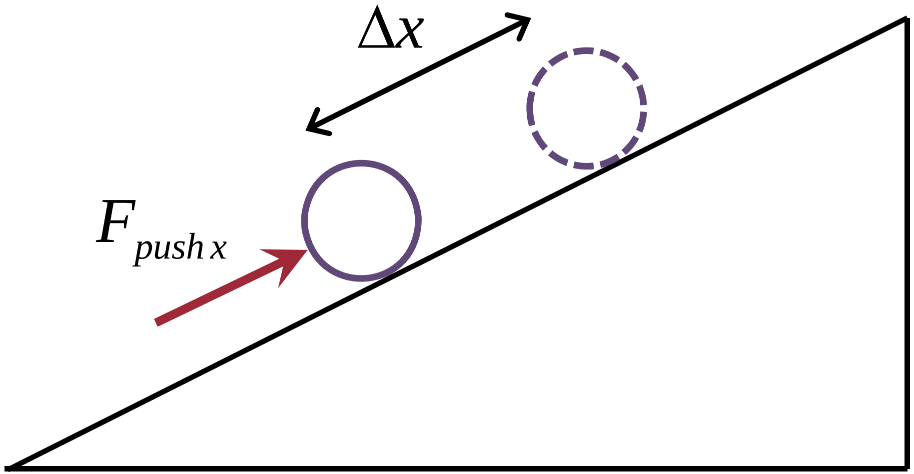
where $\mathbf{p}$ is the momentum

Force and potential energy



- We can express the same idea by thinking of a force as the gradient of a potential
- To understand this
  - suppose we are trying to change the potential energy of a ball
    - by pushing it slowly (and frictionlessly) up a hill or a slope

Force and potential energy



We get a change ΔV in potential energy V

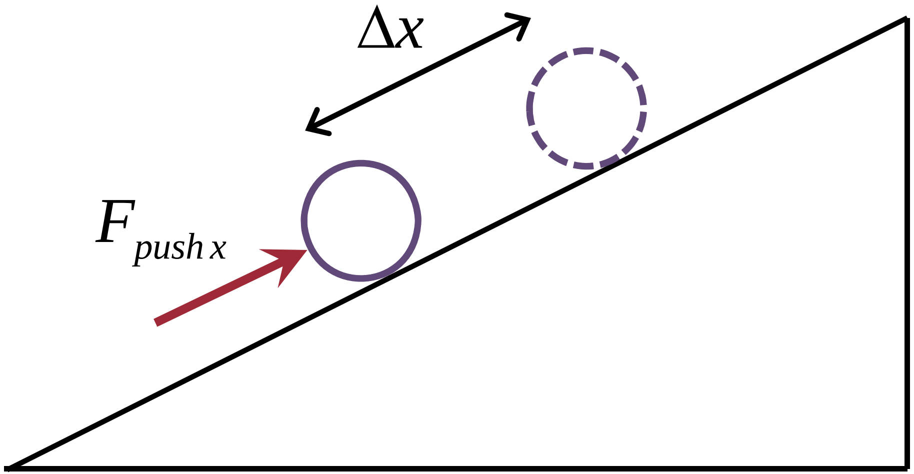
$$\Delta V = F_{pushx} \Delta x$$

by exerting a force F_{pushx}

in the x direction up the slope

through a distance Δx

Force and potential energy



Equivalently $$F_{pushx} = \frac{\Delta V}{\Delta x}$$

or in the limit $$F_{pushx} = \frac{dV}{dx}$$

The force exerted by the potential gradient on the ball is downhill

so the relation between force and potential is

$$F_x = -\frac{dV}{dx}$$

Force as a vector

We can generalize the relation between potential and force

to three dimensions

with force as a vector

by using the gradient operator

$$\mathbf{F} = -\nabla V \equiv -\left[\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k} \right]$$

Oscillations and waves 1

Modes

Modern physics for engineers

David Miller

Concepts and terminology

Concepts and terminology

"Modes" appear in many physical systems

especially those that oscillate

and in wave propagation

including classical applications in

- acoustics
- mechanical structural engineering
- the electrical engineering of waves, signals, and oscillators

Concepts and terminology

- Modes have very useful mathematical properties
  - This mathematics is at the core of quantum mechanics
- Modes are often not taught as a general concept
  - despite the wide usefulness of the idea

Concepts and terminology

- The terminology varies between fields
  - In the classical physics of oscillations or waves
    - the term “mode” is common
      - mechanical oscillating modes
        - are also often known as normal modes

Concepts and terminology

In the mathematics of modes, they are known as

eigenfunctions or basis functions

and in quantum mechanics

eigenstates or basis states, and, for atoms, orbitals

The underlying concepts are the same in all these cases

Examples of modes

Examples of modes

It is difficult to find a broad definition of modes in text books

- There is a precise mathematical answer
  - but it gives little direct physical insight
    - so we postpone it

Examples of modes

Systems showing a simple oscillating mode include

- a mass on a spring
- a pendulum
- a wine bottle “resonator” (Helmholtz resonator)

also in string instruments and loudspeaker design

Helmholtz resonators – a “wine bottle”

Helmholtz resonators – an ocarina

Examples of modes

A guitar string has a fundamental mode, and also harmonics

- as do wind instruments such as the flute

These are “standing wave” resonators

“Standing wave” resonators – a guitar string

“Standing wave” resonators – a flute or “penny whistle”

Examples of modes

- More two-dimensional bodies, such as a gong or a cymbal
  - or three-dimensional structures, such as a bell or a girder bridge
    - have a complicated set of modes
- There are also electromagnetic resonators
  - metal cavities, with specific resonant modes
    - and optical resonant cavities, formed with mirrors

Examples of modes

- Oscillating modes or resonances have at least two common features
  - First feature
    - Each resonance or mode corresponds to a distinct way in which the object oscillates
      - the pattern of oscillation is unique to a given mode
        - the way in which the patterns of oscillation are different from one another is quite specific mathematically

Examples of modes

Second feature

- For each such resonance or mode, there is one well defined numerical quantity associated with it
  - usually the frequency

At least for “loss-less” and “small amplitude” idealizations of the mode

- once excited the oscillation would stay exactly the same form
  - and everything that was oscillating would be oscillating at the same frequency in that mode

Examples of modes

- There are also “propagating modes” that arise with waves
  - One notion essentially is that the wave stays the same shape as it moves
    - Essentially then, every point that propagates propagates with the same “phase velocity”

Oscillations and waves 1

The simple harmonic oscillator

Modern physics for engineers

David Miller

Simple oscillators

- All sorts of things vibrate, such as
  - musical instruments and loudspeakers
- Many electronic circuits oscillate, such as
  - the devices that drive microwave ovens
  - or that create the radio waves in many wireless remote controls

Simple oscillators

A classic and simple example is

- the so-called simple harmonic oscillator
  - which is the kind of oscillator we get if we have
    - a mass on a spring

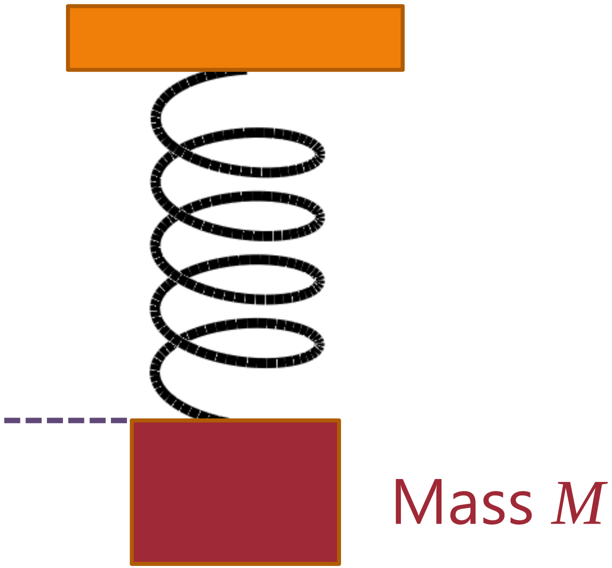
Mass on a spring

Perhaps the simplest system we can think of to illustrate modes is a simple harmonic oscillator

- consisting of a mass on a spring

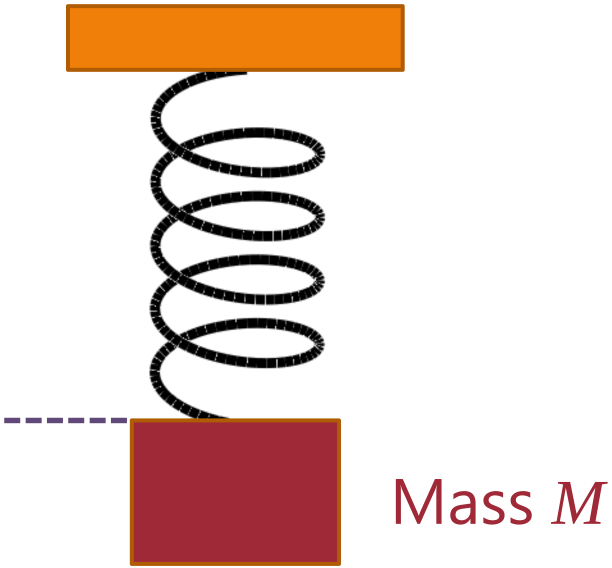
We imagine the mass can only move up and down, not sideways

- we presume it cannot twist or rotate in any way
  - and we presume that there is no friction



Mass on a spring

- The common dictionary definition of "mode" that is relevant here is that of a "manner" or "way"
  - There is only one "way" in which this mass can move
    - which is to oscillate up and down with a specific frequency



Mass on a spring

A simple spring will have a restoring force F acting on the mass M

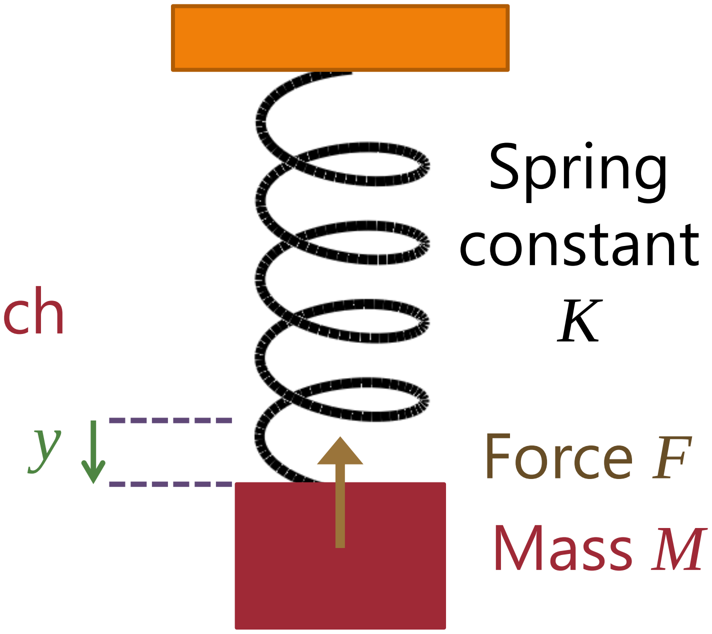
- proportional to the amount y by which it is stretched
  - For some "spring constant" K

$$F = -Ky$$

The minus sign is because this is "restoring"

- It is trying to pull y back towards zero

This gives a "simple harmonic oscillator"



Mass on a spring

From Newton's second law

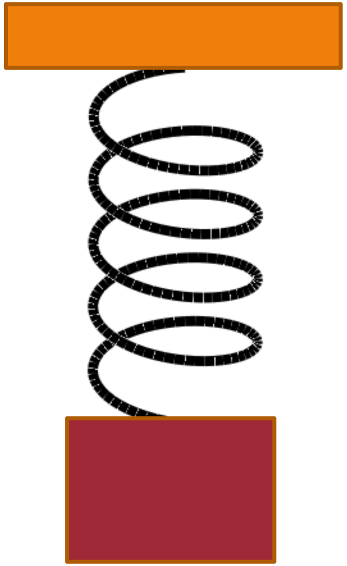
$$F = Ma = M\frac{d^2y}{dt^2} = -Ky$$

i.e., $$\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$$

where we define $\omega^2 = K/M$

we have oscillatory solutions of angular frequency $\omega = \sqrt{K/M}$

e.g., $$y \propto \sin \omega t$$



angular frequency ω, in "radians/second" = $2\pi f$ where f is frequency in Hz

Simple harmonic oscillator

A physical system described by an equation like

$$\frac{d^2 y}{dt^2} = -\omega^2 y$$

is called a simple harmonic oscillator

Many examples exist

- mass on a spring
  - in many different forms
- electrical resonant circuits
- "Helmholtz" resonators in acoustics
- linear oscillators generally