Light and quantum mechanics 1

Light, the photoelectric effect, and the photon

Modern physics for engineers

David Miller

Light and quantum mechanics

Light and quantum mechanics

With both quantum mechanics and thermal distributions

- we can explain the main behaviors of light
  - and how it interacts with matter
    - including emission and absorption
  - and the relation between them

Light and quantum mechanics

So we will be able to understand practical issues such as

- the limitations of conventional light bulbs and
- the light from the sun

and so we can introduce concepts such as

- the stimulated emission that makes lasers work

Light and quantum mechanics

- we derive Planck's distribution and
  - consequences for light spectra
- we explain key concepts in the thermodynamics of light
  - including black-body spectra and
    - Kirchhoff's law of radiation
- we derive Einstein's "A and B coefficient" argument
  - relating emission and absorption
    - including stimulated emission

The photoelectric effect and the photon

Photoelectric effect

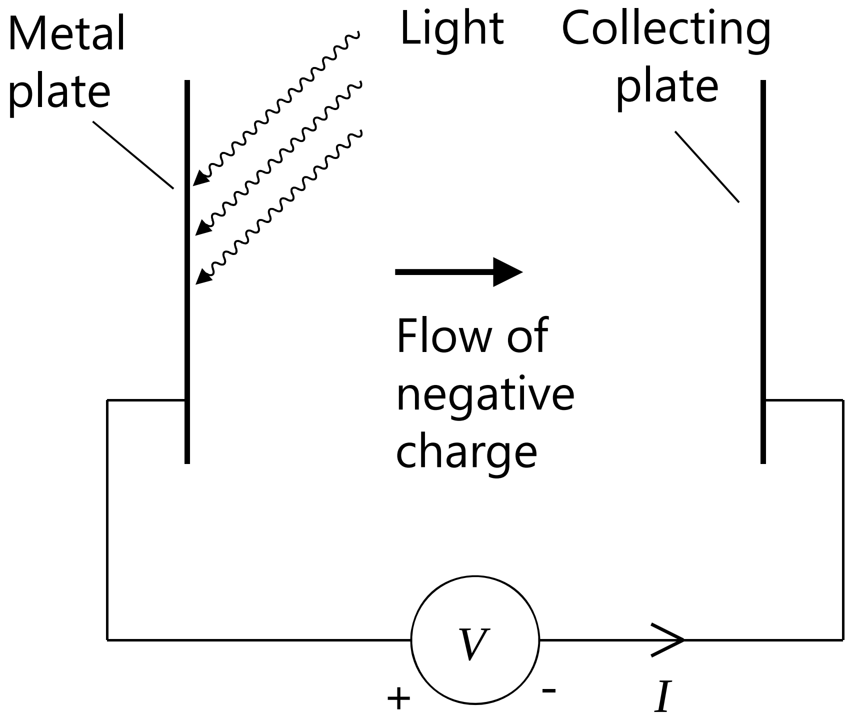
Shining ultraviolet light on the metal plate

gives a flow of negative charge (Hertz, 1887)

The flow can be stopped with a specific voltage

which is independent of the brightness

and dependent only on the frequency (Lenard, 1902)



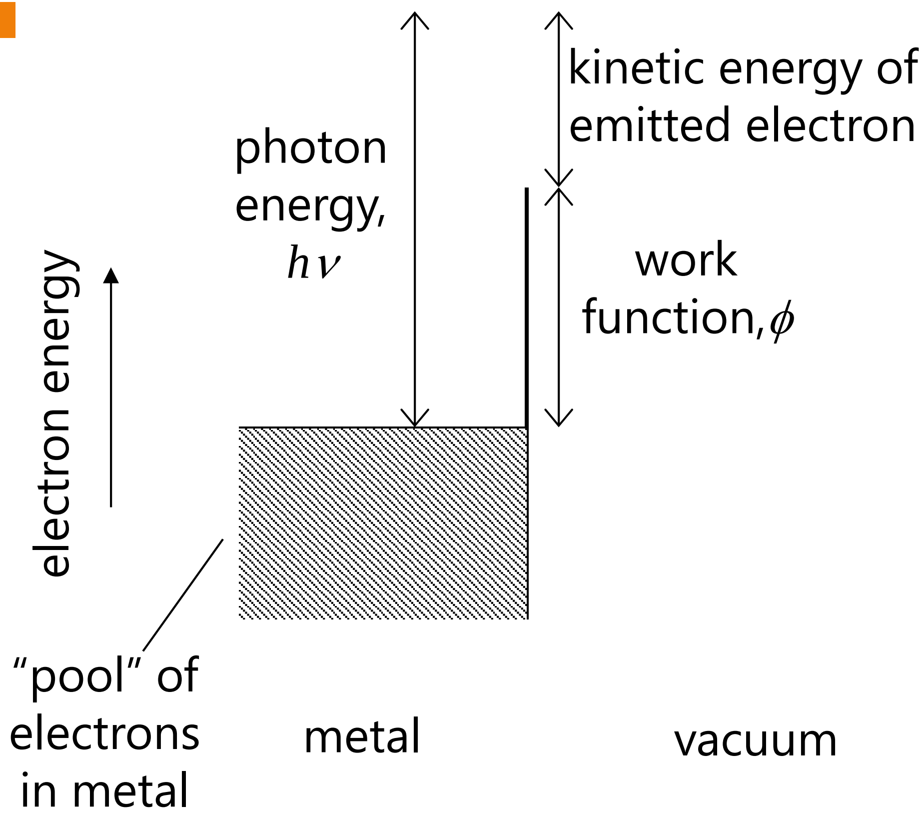
Photoelectric effect

Einstein's proposal (1905)

- light is actually made up out of particles
  - photons, of energy $E = h\nu$

The kinetic energy of the emitted electrons

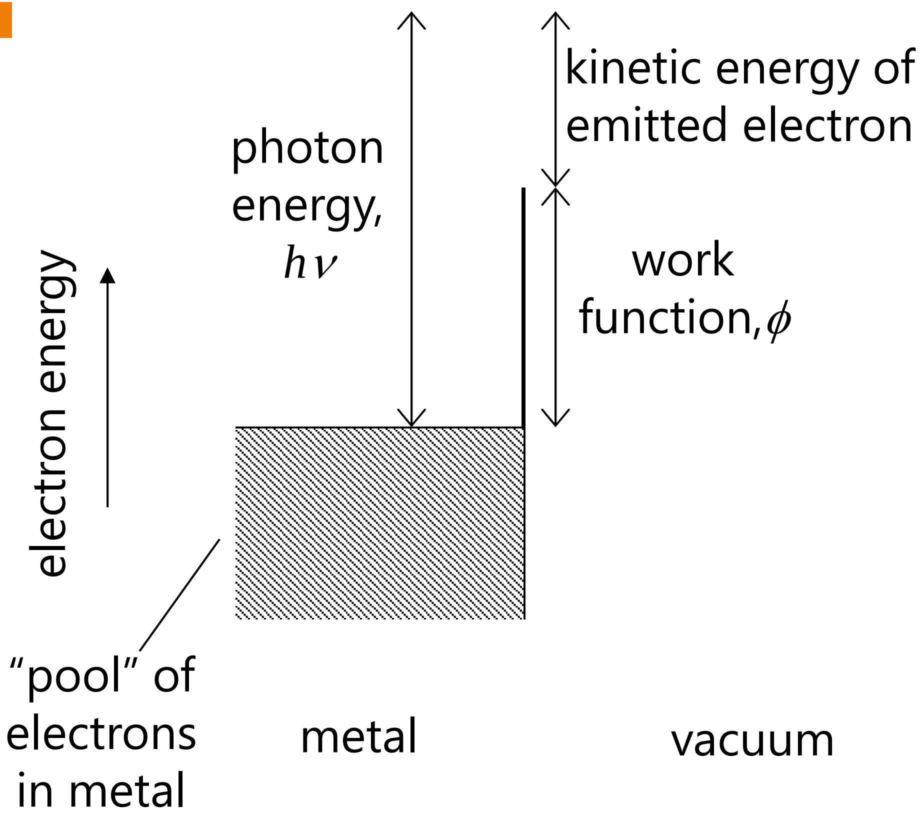
- is the energy left over after
  - the electron has been "lifted" over the work function barrier



Photoelectric effect

So, the electrons start out with an "excess" kinetic energy

$h\nu - \phi$ (the work function)



Photoelectric effect

So, the electrons start out with an "excess" kinetic energy

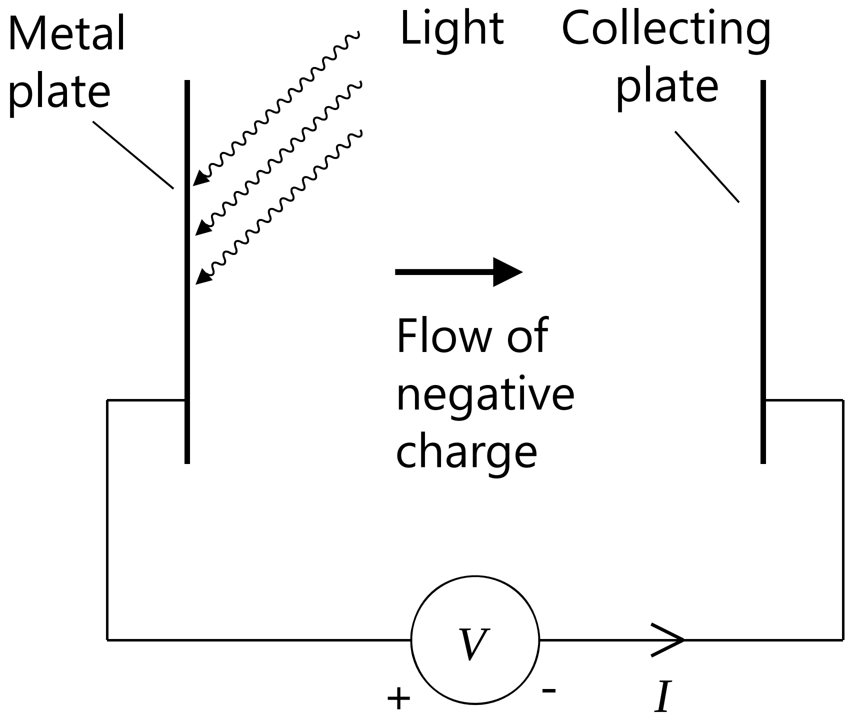
$h\nu - \phi$ (the work function)

The voltage that just stops the current

is the one that just stops all the electrons

with this kinetic energy

from managing to reach the collecting plate



Light and quantum mechanics 1

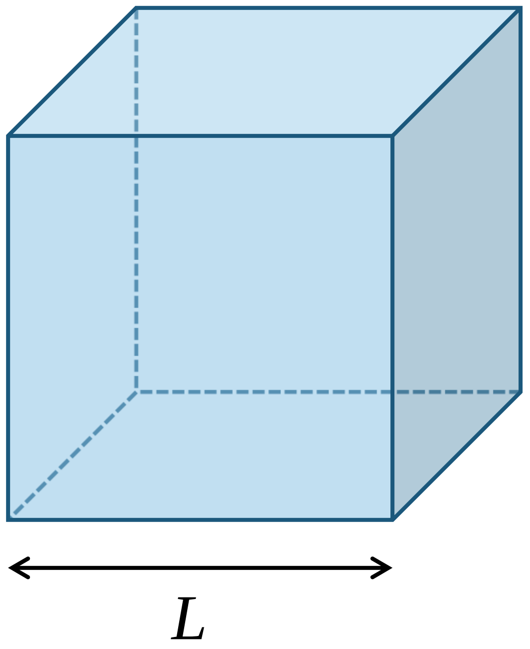
Light and modes

Modern physics for engineers

David Miller

Light and modes

- To understand light and quantum mechanics
  - in addition to photons, we need to
    - consider modes for light
- To do this, we imagine a cubic cavity of side L
  - with perfectly reflecting walls
    - and consider its modes
- From this we get the number of modes per unit volume



Light and modes

The possible modes of the cavity are those for which

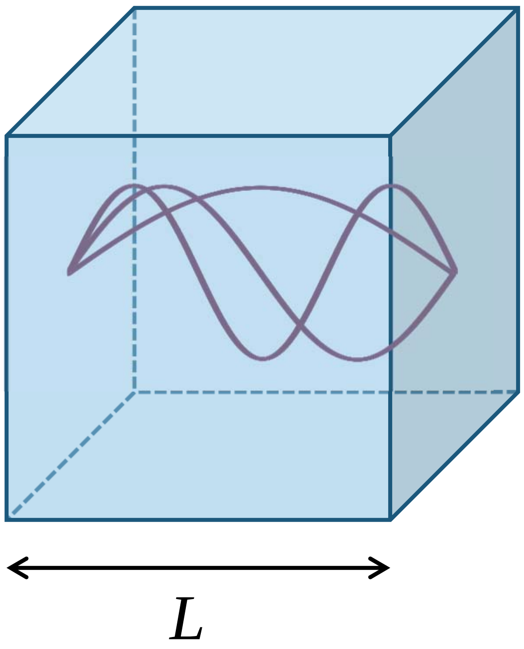
the wave reaches zero at the walls

hence the possible modes are sine waves

in all 3 directions at once

$$\sin(k_x x)\sin(k_y y)\sin(k_z z)$$

in which integer numbers of half waves fit within the cavity



Light and modes

For such a wave

$$\sin(k_x x)\sin(k_y y)\sin(k_z z)$$

to be zero at the walls

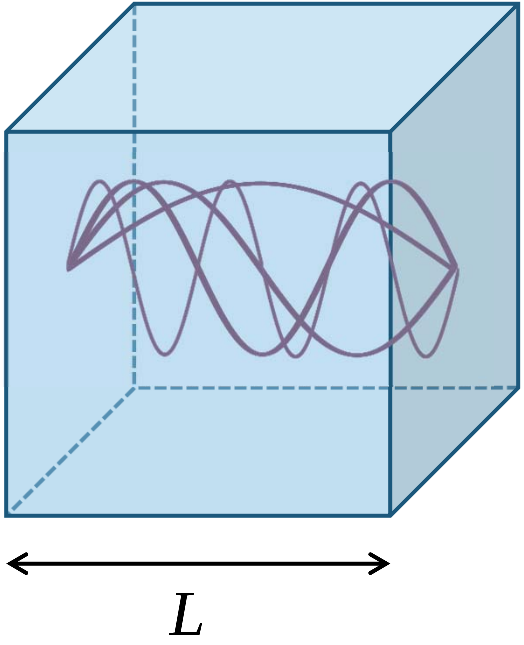
the allowed values of k_x

the wavevector magnitude in the x direction

are $k_x = \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \ldots, \frac{n_x \pi}{L}, \ldots$

where n_x is an integer

and similarly for the y and z directions



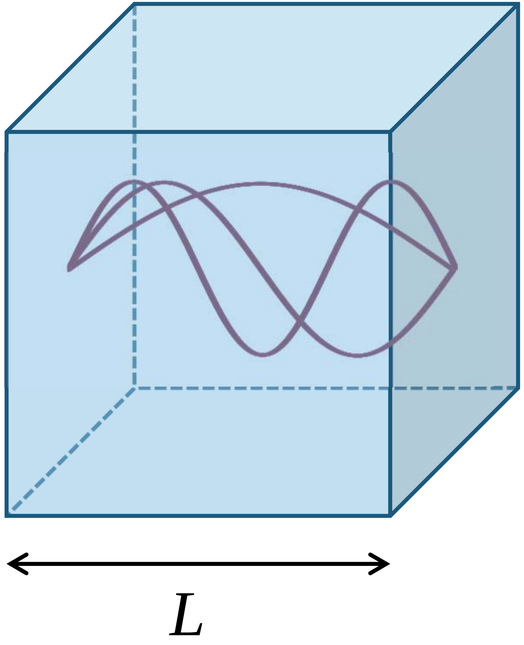
Light and modes

Note that, with

$$k_x = \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \ldots, \frac{n_x \pi}{L}, \ldots$$

and similarly for the y and z directions

the allowed values for each component k_x, k_y, and k_z are spaced by π / L



Light and modes

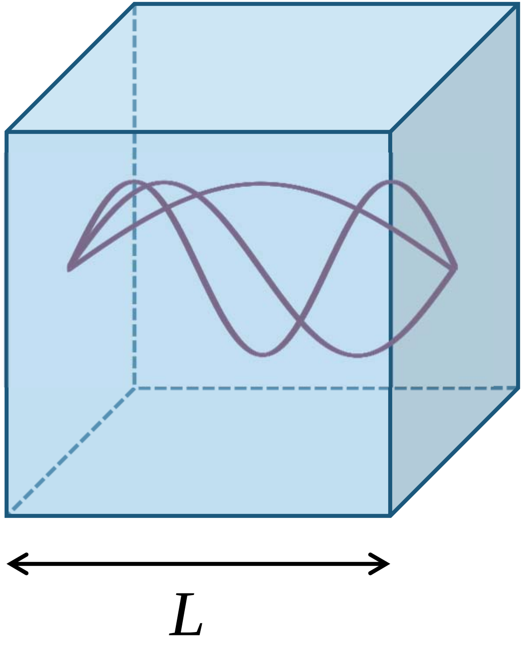
So, if we are interested in some range of k_x of size Δk_x

we should expect to find

$$\frac{\Delta k_x}{(\pi / L)} = \frac{L}{\pi} \Delta k_x$$

different possible k_x values in that range

We can argue similarly for k_y and k_z



Light and modes

So, for ranges Δk_x, Δk_y, and Δk_z

we should expect to find

$$\frac{L}{\pi}\Delta k_x \frac{L}{\pi}\Delta k_y \frac{L}{\pi}\Delta k_z = \frac{L^3}{\pi^3}\Delta k_x \Delta k_y \Delta k_z$$

$$\equiv g_{\mathbf{k}} \Delta k_x \Delta k_y \Delta k_z$$

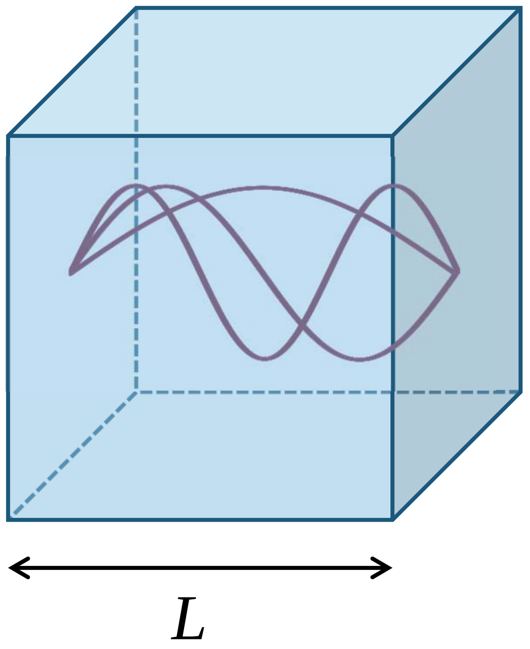
different possible values of

$$\mathbf{k} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}}$$

Here we can call $g_{\mathbf{k}} \equiv L^3 / \pi^3 = V / \pi^3$

the density of states in k-space

for a volume $V = L^3$



Light and quantum mechanics 1

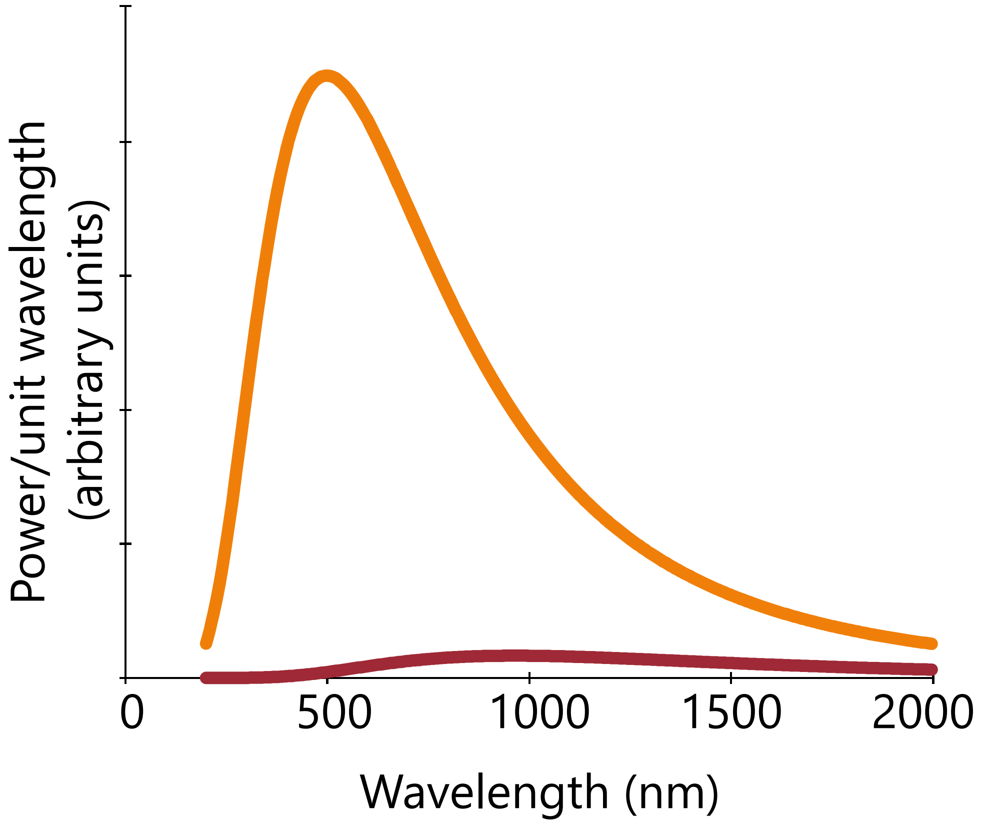
Thermal radiation

Modern physics for engineers

David Miller

Black-body spectrum

- We remember the
  - "black-body spectrum"
    - from a hot, black body
  - The output power (per unit wavelength)
  - for a black body at 5800K
    - approximately like the sun
  - for a black body at 3000K
    - approximately like an incandescent light bulb



Planck distribution

We remember the Planck distribution

- the average number of photons per mode in thermal equilibrium at temperature T is
  - the Planck distribution

$$\langle q \rangle = \frac{1}{\exp\left(\hbar\omega / k_B T\right) - 1}$$

(We saw that the Planck distribution is

- a Bose-Einstein distribution
  - with chemical potential of zero)

Note incidentally that $\langle q \rangle$ need not be an integer

Thermal radiation

With $\langle q \rangle = 1/\left[\exp\left(\hbar\omega / k_B T\right) - 1\right]$ photons

in a given mode of angular frequency ω

the total energy in the field per unit volume

the energy density U, is

$$U = \frac{1}{V}\sum_{polarizations}\sum_{k_x}\sum_{k_y}\sum_{k_z}\langle q \rangle \hbar\omega = \frac{2}{V}\sum_{k_x}\sum_{k_y}\sum_{k_z}\langle q \rangle \hbar\omega$$

where we have summed over the two polarizations

which gives the factor of 2

Thermal radiation

We presume the volume or "box" we are considering is large

so the allowed values of the components k_x, k_y, and k_z

are very closely spaced

So we can approximate the sum by an integral

where we allow each component k_x, k_y, and k_z

to range from approximately zero to infinity

and we use the density of states $g_{\mathbf{k}} = V / \pi^3$

So the total energy per unit volume for all these modes is

$$U = \frac{2}{V} \sum_{k_x} \sum_{k_y} \sum_{k_z} \langle q \rangle \hbar\omega \simeq \frac{2}{V} \int_{k_x=0}^{\infty} \int_{k_y=0}^{\infty} \int_{k_z=0}^{\infty} \langle q \rangle \hbar\omega \, g_{\mathbf{k}} \, dk_x dk_y dk_z$$

Thermal radiation

Nothing in this integral $U \simeq \frac{2}{V}\int_{k_x=0}^{\infty}\int_{k_y=0}^{\infty}\int_{k_z=0}^{\infty}\langle q\rangle \hbar\omega\, g_{\mathbf{k}}\, dk_x dk_y dk_z$

depends on the direction of the wave vector $\mathbf{k}$

so we usefully change to spherical polar coordinates

We can integrate in spherical shells of radius k

and surface area $4\pi k^2$ and thickness dk

though dividing by 8 to consider only

the one octant that corresponds to

positive k in each direction

Thermal radiation

So with $g_{\mathbf{k}} = V / \pi^3$ we have

$$U \simeq \frac{2}{8V} \int_{k=0}^{\infty} \langle q \rangle \hbar\omega \, g_{\mathbf{k}} \, 4\pi k^2 dk = \int_{k=0}^{\infty} \langle q \rangle \hbar\omega \frac{1}{\pi^2} k^2 dk$$

For light in free space, for angular frequency ω

$$k = \omega / c$$

(this is an equivalent relation to $f = c / \lambda$ between frequency f, wavelength λ, and velocity of light c)

so changing variables from k to ω, we have

$$U \simeq \frac{\hbar}{\pi^2 c^3} \int_{\omega=0}^{\infty} \omega^3 \langle q \rangle d\omega$$

Thermal radiation

We can choose to think of this integral $U \simeq \frac{\hbar}{\pi^2 c^3} \int_{\omega=0}^{\infty} \omega^3 \langle q \rangle d\omega$

as an integral over

a quantity u_ω, i.e., $U = \int_0^\infty u_\omega d\omega$

Then u_ω is the energy density per unit (angular) frequency

for light in thermal equilibrium at a temperature T

We can write this explicitly as

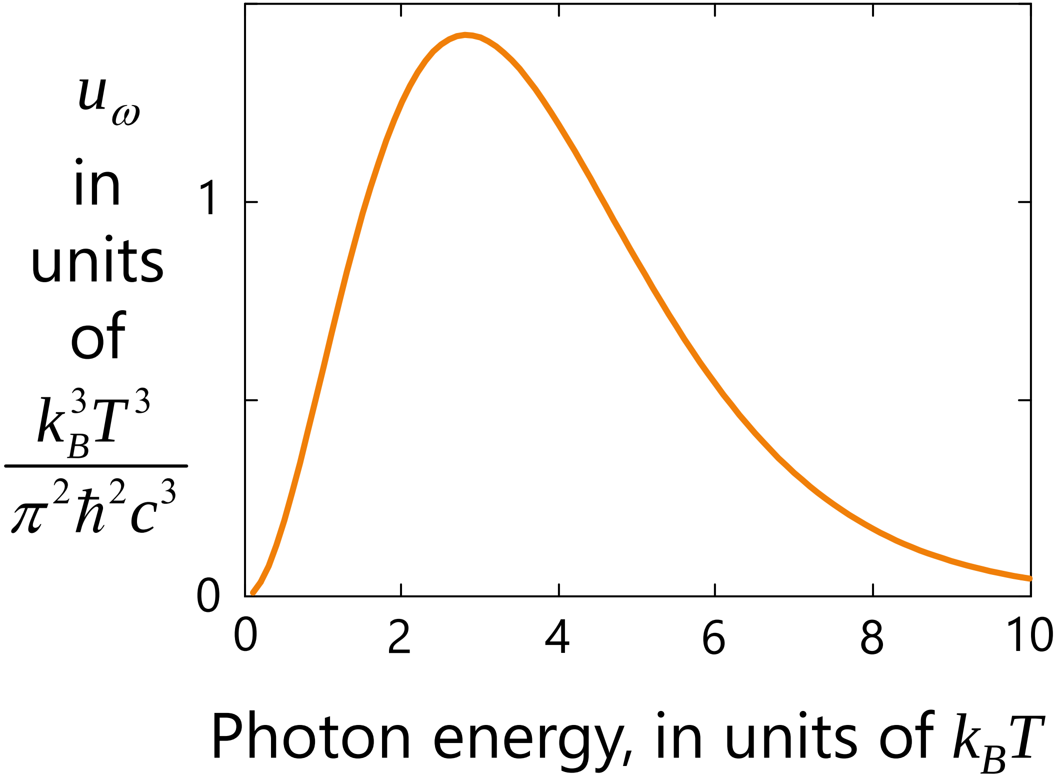
$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp(\hbar\omega / k_B T) - 1}$$

Planck and Stefan-Boltzmann radiation laws

Planck radiation law

$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp\left(\hbar\omega / k_B T\right) - 1}$$

- We plot this spectrum
  - against photon energy
- This is the black-body spectrum
  - though we need one more step to get there completely
- The exponential cuts off high frequencies
  - avoiding the ultraviolet catastrophe



Stefan-Boltzmann law

We can now complete the integration over frequency to get the total thermal radiation density

$$U = \int_0^\infty u_\omega d\omega = \int_0^\infty \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp\left(\hbar\omega / k_B T\right) - 1} d\omega$$

$$= \frac{k_B^4 T^4}{\pi^2 \hbar^2 c^3} \int_0^\infty \frac{x^3}{\exp\left(x\right) - 1} dx$$

where we have changed variables to $x = \frac{\hbar\omega}{k_B T}$

We note the result $\int_0^\infty \frac{x^3}{\exp\left(x\right) - 1} dx = \frac{\pi^4}{15}$

Stefan-Boltzmann law

Hence, for the energy per unit volume inside the cavity

we obtain the

Stefan-Boltzmann law $U = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T^4$

In an electromagnetic field

or equivalently, in a "photon gas"

at a given temperature

the energy density is proportional to the fourth power of the temperature