Light and quantum mechanics 2

Black body radiation and Kirchhoff's law

Modern physics for engineers

David Miller

Black body radiation

Black body radiation

Suppose we have a cavity for light

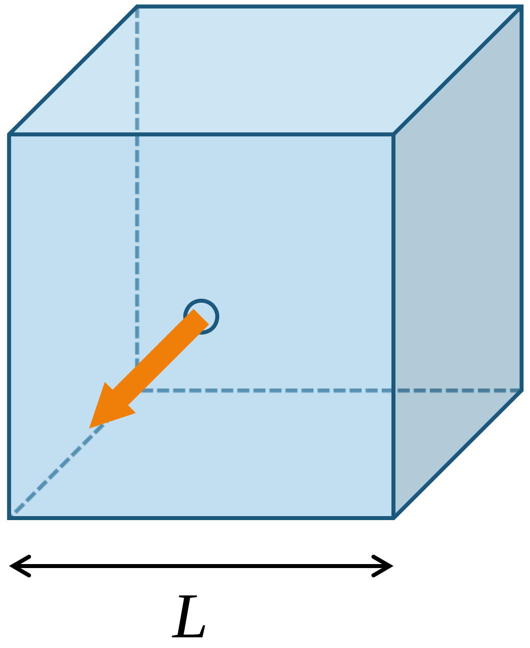
and we make a small hole in it

so some of the radiation can escape

We can make this hole as small as we wish

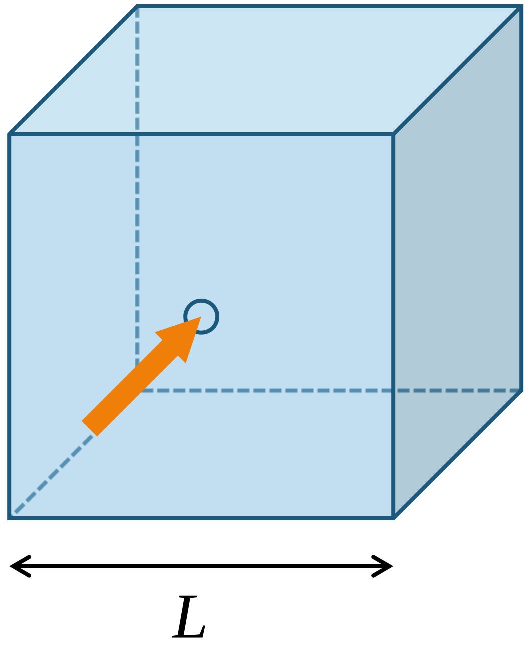
to make this perturbation of the system as small as we want

So the spectrum of this radiation will be the same as in the cavity



Black body radiation

- Suppose now we shine light into this body through this small hole
  - Because the hole is small
    - the light will bounce around many times inside the body
- Assuming at least some small absorption inside the cavity
  - this light will be absorbed and essentially not re-emitted through the hole



Black body radiation

Such an object is known as a "black-body" because

it is a perfect absorber of light

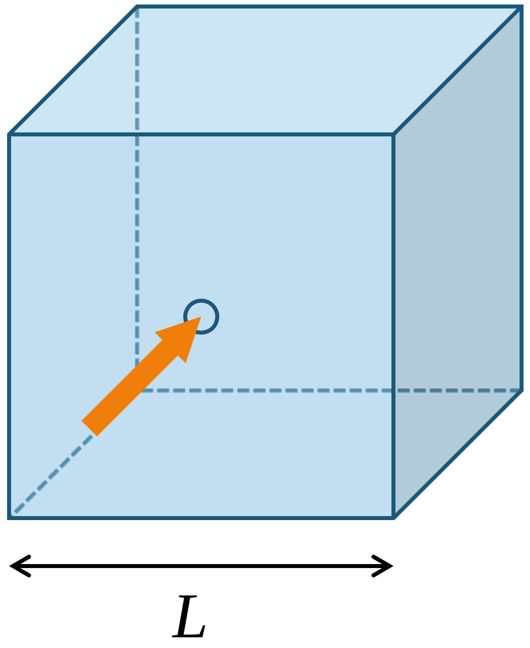
Because this is a cavity at thermal equilibrium

the light inside has the spectrum given by Planck's radiation law, so

the light emitted through the hole

called "black body radiation"

has this spectrum also



Black body radiation

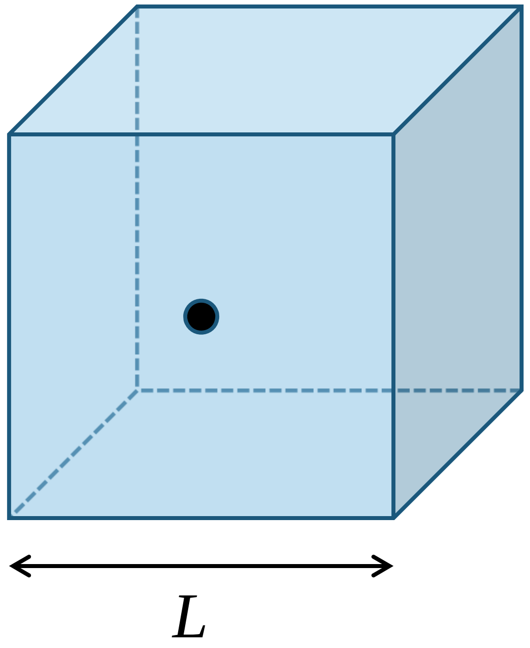
Suppose now we close off this hole with a "black" (perfectly absorbing) surface

In thermal equilibrium

the radiation emitted by this black surface into the cavity

must equal

the radiation landing on the black surface from the inside of the cavity



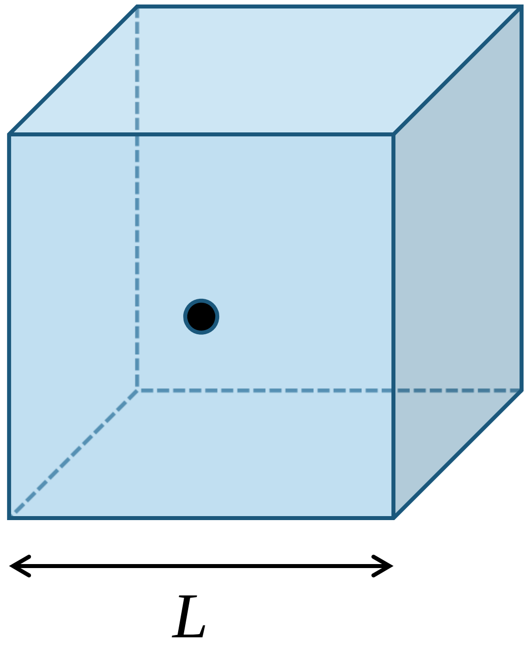
Black body radiation

Hence the thermal radiation emitted from a black surface

is equal to the thermal radiation from a small hole in a cavity (at the same temperature)

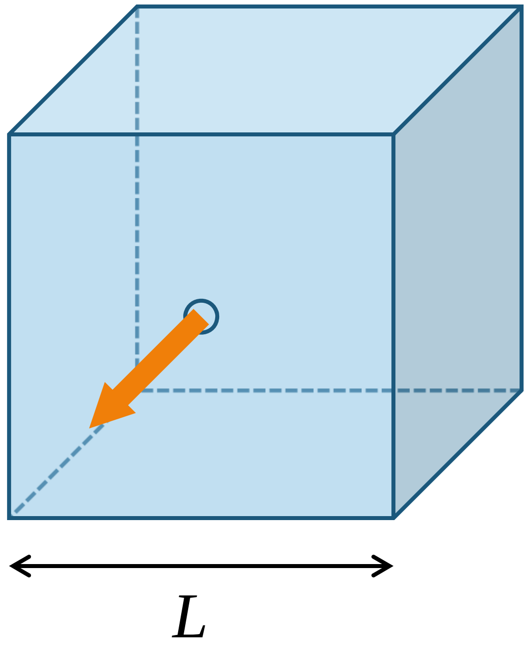
Hence in the Planck radiation law (and the related Stefan-Boltzmann law)

we have essentially calculated the form of the thermal radiation emitted from a black surface



Black body radiation

- No body in thermal equilibrium emits more radiation than this black body
- If there were such a body
  - we could heat up the cavity solely with the radiated thermal energy from that body
    - with the body at the same temperature as the cavity
      - which would violate the second law of thermodynamics



Bad news for light bulbs

- There is a limit to how much light we can get out just by heating something up
  - and there is nothing we can do about this limit
    - If we want more light from a given size of incandescent bulb
      - we have to make it hotter
        - and materials can only be made so hot before they evaporate

Kirchhoff’s law of radiation

Kirchhoff's law of radiation

Consider now a "non-black" object

- that emits less thermal radiation
  - about some given wavelength
  - than a black body

The ratio of its emitted energy to that of a black body is called

- the "emissivity", e_s
  - which can be a function of
    - wavelength
  - and $e_s \leq 1$

Kirchhoff’s law of radiation

Suppose now that the object absorbs a fraction

a, the “absorptivity”

of the incident radiation at or about the wavelength of interest

Kirchhoff's law of radiation

But, in thermal equilibrium

- the body must emit the same amount of radiation that it absorbs
  - otherwise we could heat the body up or cool the body down
    - with black body radiation from a body at the same temperature
      - violating the second law of thermodynamics

Kirchhoff's law of radiation

Hence

Kirchhoff's law

the absorptivity and emissivity are equal

Incidentally, this means that a good reflector emits very little thermal radiation

For a recent discussion, see also PNAS **114**, 4336 (2017) doi:10.1073/pnas.1701606114

Light and quantum mechanics 2

Einstein's A and B coefficient argument

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Planck radiation law

Previously we derived the Planck radiation law

the energy per unit volume per unit (angular) frequency

in a radiation field near (angular) frequency ω

$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp(\hbar\omega / k_B T) - 1}$$

Now we will rewrite this in terms of frequency ν

giving the energy per unit volume per unit frequency

which we will write as

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp(h\nu / k_B T) - 1}$$

Atom with two states

Suppose we have atoms with two possible states

state 1 of energy E_1

state 2 of (greater) energy E_2

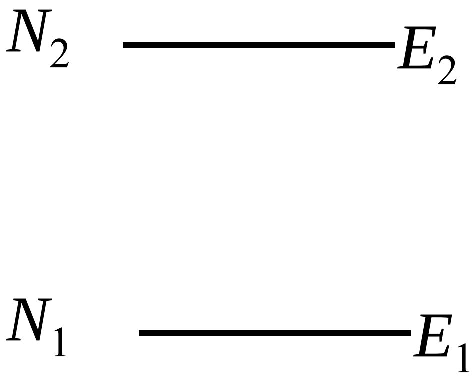
We presume a collection of atoms

N_1 (per unit volume) in state 1

N_2 (per unit volume) in state 2

with everything, atoms and radiation

in thermal equilibrium at temperature T



Boltzmann factor

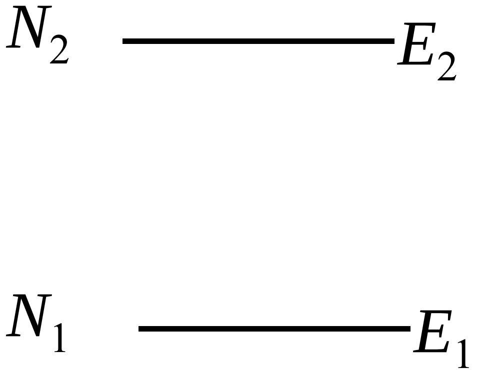
We also expect in thermal equilibrium that

the ratio between N_2 and N_1

should be given by the Boltzmann factor

so that

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right)$$



Emission and absorption

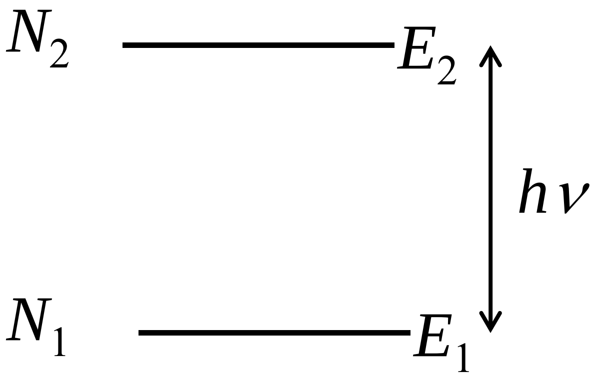
We presume the only way atoms change states from 2 to 1 is by emitting a photon of energy

$$h\nu = E_2 - E_1$$

and the only way to change states from 1 to 2 is by absorbing such a photon

We can also rewrite the Boltzmann factor as

$$\frac{N_2}{N_1} = \exp\left(-\frac{h\nu}{k_B T}\right)$$



Spontaneous emission

We expect there is an emission process

spontaneous emission

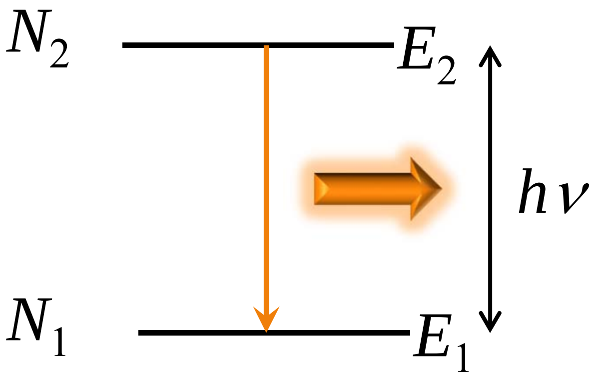
by which an atom in state 2

will emit a photon

and fall to state 1

We expect this process happens

independent of the amount of light also present



Spontaneous emission rate

We propose the number of such spontaneous emission transitions

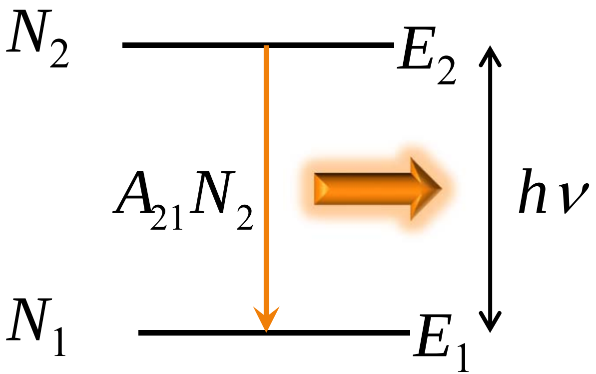
is simply proportional to the number of atoms in state N_2

and we propose a constant coefficient A_{21}

an intrinsic property of the atom

not dependent on temperature

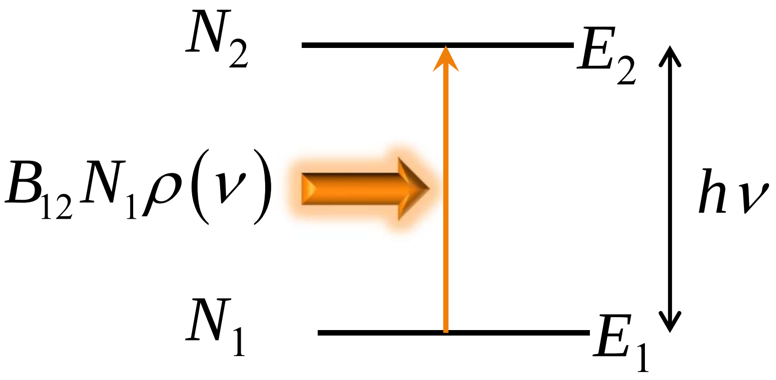
giving a spontaneous emission rate $A_{21}N_2$



Absorption rate

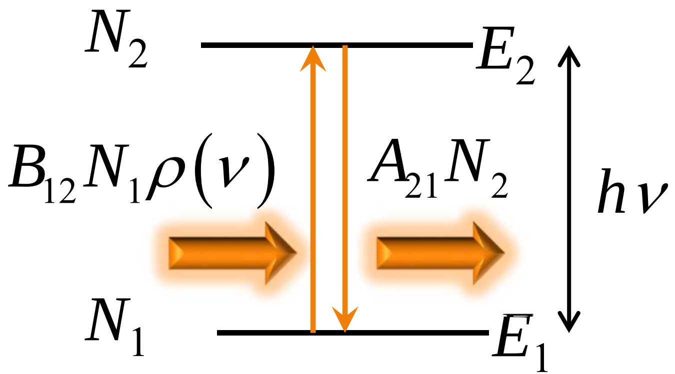
We propose an absorption rate from the N_1 atoms in state 1 to state 2

- $\propto \rho(\nu)$, the energy density in the radiation field near frequency ν
  - with a constant coefficient B_{12}
    - an intrinsic temperature-independent atomic property
      - so we have a total absorption rate $B_{12}N_1\rho(\nu)$



Detailed balance

- A basic presumption of statistical mechanics is that we have
  - detailed balance
    - In equilibrium
      - we must have exactly equal rates for processes in both directions
- But this gives a problem if we stop our model here



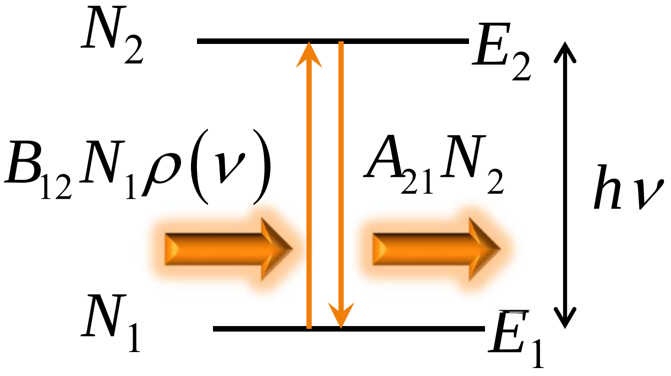
Detailed balance problem

If we ask for detailed balance and equate

$$A_{21} N_2 = B_{12} N_1 \rho(\nu)$$

i.e.,

$$\frac{A_{21}}{B_{12}} = \frac{N_1}{N_2}\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{\exp(h\nu / k_B T)}{\exp(h\nu / k_B T)-1}$$



the left side does not depend on temperature

but the right side does

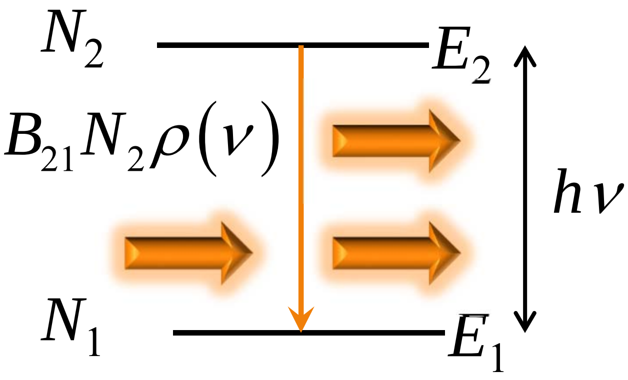
Stimulated emission

Einstein realized the way out

- Propose another process
  - stimulated emission
    - just like absorption
      - but from state 2 to state 1
        - an additional emission rate

$$B_{21} N_2 \rho(\nu)$$

This coefficient B_{21} is proposed to also be an intrinsic temperature-independent atomic property



Einstein A and B coefficient argument

emission / absorption

Now detailed balance gives $\underbrace{A_{21}N_2 + B_{21}N_2\rho(\nu)}_{\text{emission}} = \underbrace{B_{12}N_1\rho(\nu)}_{\text{absorption}}$

i.e., $A_{21} = \left(B_{12}\frac{N_1}{N_2} - B_{21}\right)\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{\left[B_{12}\exp(h\nu/k_{BT}) - B_{21}\right]}{\exp(h\nu/k_B T) - 1}$

So if we choose $B_{21} = B_{12}$

then we can have detailed balance

and we also obtain $A_{21} = \frac{8\pi h\nu^3}{c^3}B_{12}$

Einstein A and B coefficient argument

So, if we presume intrinsic microscopic processes for

spontaneous emission $A_{21}N_2$

and absorption $B_{12}N_1\rho(\nu)$

then to achieve detailed balance

we need to propose stimulated emission $B_{21}N_2\rho(\nu)$

Then we find $$A_{21} = \frac{8\pi h\nu^3}{c^3}B_{12} \qquad B_{21} = B_{12}$$

also implying the same microscopic atomic coefficient underlies all three processes

For a recent update, see Adv. Opt. Photon. 11, 679 (2019) DOI: 10.1364/AOP.11.000679

Spontaneous and stimulated emission

Stimulated emission

We can rewrite our stimulated emission rate $B_{21} N_2 \rho(\nu)$

using the thermal energy density per unit frequency

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp(h\nu / k_B T) - 1}$$

as $$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} n_{ph}$$

where n_{ph} is the number of photons per mode

which is, thermally, $n_{ph} = \dfrac{1}{\exp(h\nu / k_B T) - 1}$

Ratio of stimulated to spontaneous emission

The total spontaneous emission rate per unit volume per unit frequency is $R_{sp} = A_{21} N_2$

The total stimulated emission rate per unit volume per unit frequency is $R_{stim} = B_{21} N_2 \left(8\pi h\nu^3 / c^3\right) n_{ph}$

and we know that $A_{21} = \left(8\pi h\nu^3 / c^3\right) B_{21}$

So $$\frac{R_{stim}}{R_{sp}} = \frac{B_{21} N_2 \left(8\pi h\nu^3 / c^3\right)}{A_{21} N_2} n_{ph} = \frac{B_{12} N_2 \left(8\pi h\nu^3 / c^3\right)}{\left(8\pi h\nu^3 / c^3\right) B_{12} N_2} n_{ph} = n_{ph}$$

The ratio of stimulated to spontaneous emission in a given mode

is the number of photons in the mode

Ratio of stimulated to spontaneous emission

For visible radiation, e.g., of photon energy ~ 2.5 eV

number of photons per mode at 5800 K (sun temperature) directly from the sun is

$$\frac{1}{\exp(h\nu / k_B T) - 1} = \frac{1}{\exp(2.5e / k_B T) - 1} \simeq \exp\left(-2.5\frac{e}{k_B T}\right)$$

$$\simeq \exp\left(-2.5\frac{1.602\times10^{-19}}{1.38\times10^{-23}\times5800}\right) \simeq \exp(-5) = 0.007$$

so stimulated emission is mostly negligible in normal visible light even from the sun

Stimulated emission

But now we assert that this stimulated emission process

- is something that depends on the number of photons per mode
  - independent of whether that number is from a thermal distribution or otherwise

Hence, if we can arrange for a large number of photons per mode

- stimulated emission can dominate
  - hence the laser