The quantum view of the world 3

Waves, diffraction and uncertainty

Modern physics for engineers

David Miller

An uncertainty principle

Diffraction from an aperture

One well-known wave phenomenon

is diffraction from an aperture

For light of wavelength λ

incident on an aperture of size d

at large distances the resulting waves spread out

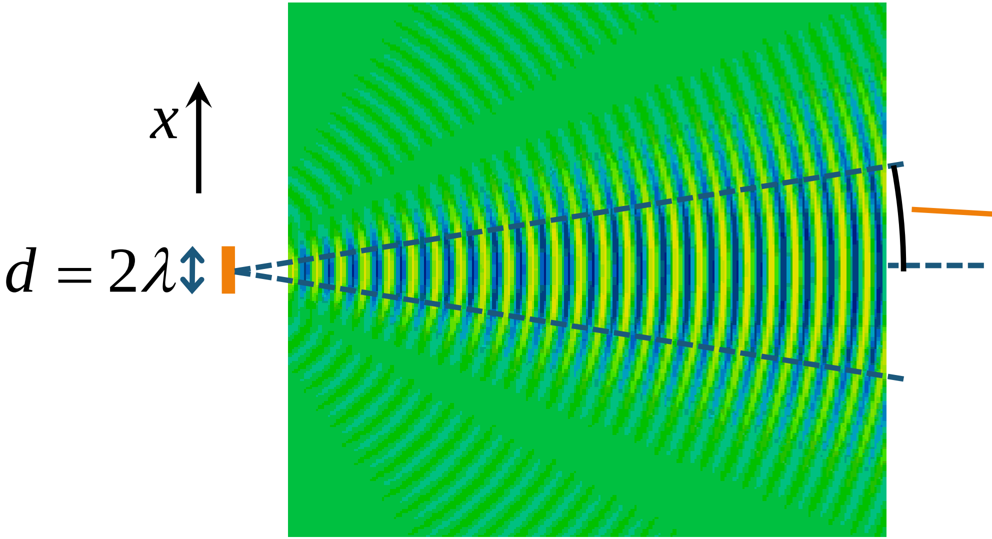
with a spreading angle loosely of order $\theta \sim \lambda / d$

For our arguments below

we choose a characteristic “half angle”

$$\theta_c \sim \frac{\lambda}{2\pi d}$$

Diffraction angle



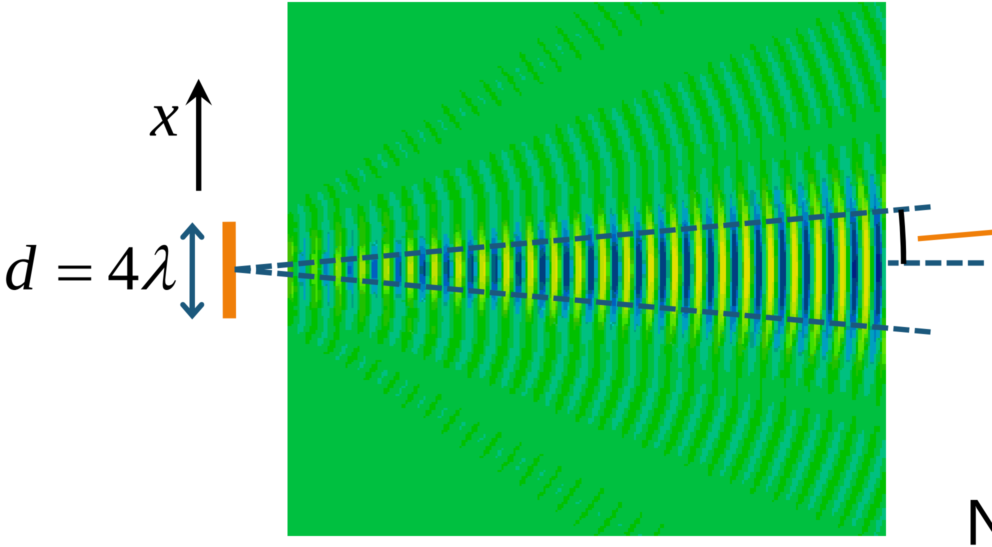
For an aperture of width d

a characteristic diffraction angle would be

$$\theta_c \sim \frac{\lambda}{2\pi d}$$

where λ is the wavelength

Diffraction angle



For an aperture of width d

a characteristic diffraction angle would be

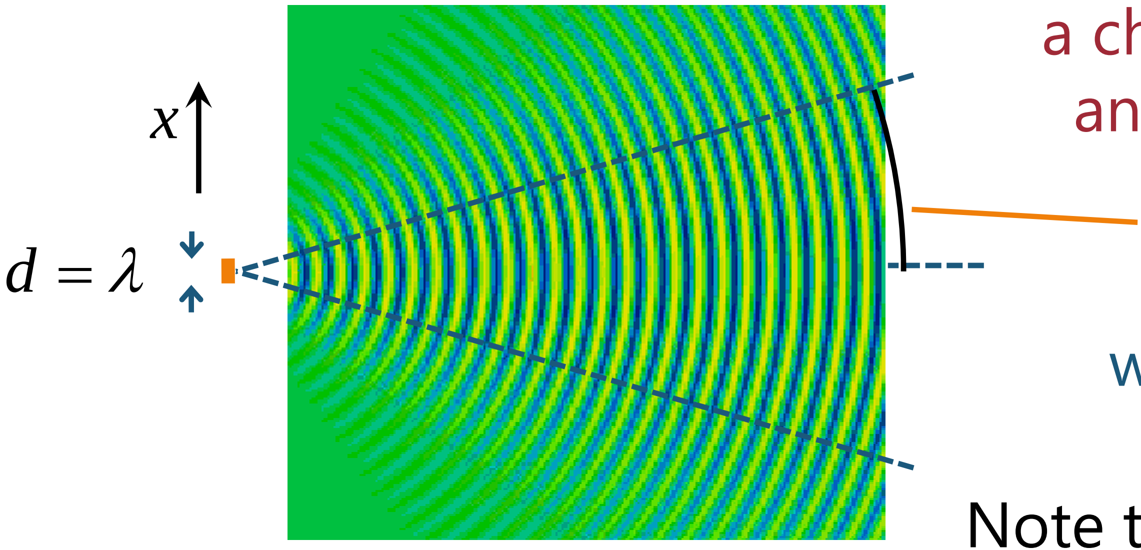
$$\theta_c \sim \frac{\lambda}{2\pi d}$$

where λ is the wavelength

Note the inverse relation

larger aperture, smaller angle

Diffraction angle



For an aperture of width d

a characteristic diffraction angle would be

$$\theta_c \sim \frac{\lambda}{2\pi d}$$

where λ is the wavelength

Note the inverse relation

larger aperture, smaller angle

smaller aperture, larger angle

Uncertainty principles in classical physics

Such an inverse relationship between

the uncertainty in one quantity and

the uncertainty in another

we can call an uncertainty principle

The relation between the uncertainty in the angular direction of the wave

the diffraction angle

and the uncertainty in the position of the wave

the width of the aperture

is a good classical example

Uncertainty principles in classical physics

It is well known in Fourier analysis of temporal signals

that we cannot simultaneously have both

a well defined frequency and

a well defined time

If a signal is a short pulse

it is necessarily made up out of a range of frequencies

$$\Delta\omega\Delta t \geq \frac{1}{2}$$

The shorter the pulse is

the larger the range of frequencies

Uncertainty principle for particles

With quantum mechanics and particles

we can think of the width of the aperture as

the uncertainty in the position in the x direction

- of a photon if it is an electromagnetic wave, or
- of an electron if it is a quantum mechanical wave

For an uncertainty in position $\pm\Delta x$ relative to the midpoint

then we can take $\Delta x = d/2$

so
$$\theta_c \sim \frac{\lambda}{2\pi d} = \frac{\lambda}{2\pi}\frac{1}{2\Delta x}$$

Uncertainty principle for particles

For wavelength λ

the magnitude of the wavevector is $k = 2\pi / \lambda$

If we consider a wavevector oriented

at up to about some angle θ_c to the horizontal

then we can consider it to have a component of the wavevector in the x direction

up to about $\Delta k_x = k \sin\theta_c \simeq k\theta_c$

at least for small angles θ_c

Hence, we have

$$\Delta k_x \simeq k\theta_c \simeq \frac{2\pi}{\lambda}\frac{\lambda}{2\pi}\frac{1}{2\Delta x} = \frac{1}{2\Delta x}$$

Uncertainty principle for particles

At least for electrons, from de Broglie's hypothesis

momentum $p = \hbar k$

so we take the uncertainty Δp_x

in the momentum in the x direction

to correspond with the uncertainty Δk_x

in the x component of the wavevector

That is, we interpret $\Delta p_x \equiv \hbar \Delta k_x$

Hence $\Delta k_x \simeq 1/2\Delta x$ leads to

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Heisenberg's uncertainty principle

We recognize this relation $\Delta p_x \Delta x \geq \frac{\hbar}{2}$

as one well-known statement of Heisenberg's uncertainty principle

We have written an inequality here

because it is certainly possibly to have uncertainties worse than this limit

for example if the incident "wave" in the aperture is not a plane wave or has some structure to it

This relation can be derived rigorously with tighter definitions and deeper mathematics

The uncertainty principle and measurement

The uncertainty principle and measurement

Given that we can view particles as waves

- then uncertainty principles follow for the same reasons as in the classical world
  - so there is nothing bizarre in saying
    - a "particle" cannot simultaneously have a well-defined position and a well-defined momentum
      - this is a natural consequence of the wave character

The uncertainty principle and measurement

When Heisenberg's uncertainty principle was introduced

the idea of quantum mechanical measurement

was mixed with these notions

of a particle not simultaneously having well-defined position and momentum

which is not itself surprising

if we view particles as having wave character

The uncertainty principle and measurement

Popular discussions of Heisenberg's uncertainty principle often state

"the act of measuring something changes it"

but that is really a separate problem of quantum mechanical measurement

which is a much more difficult issue

known as the "measurement problem"

and is associated with the "collapse of the wavefunction"

The quantum view of the world 3

Diffraction by two slits – Young's slits

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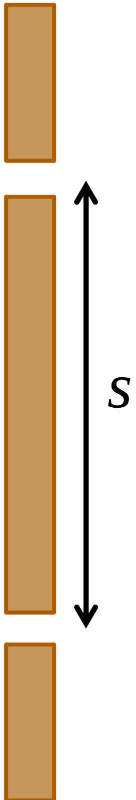
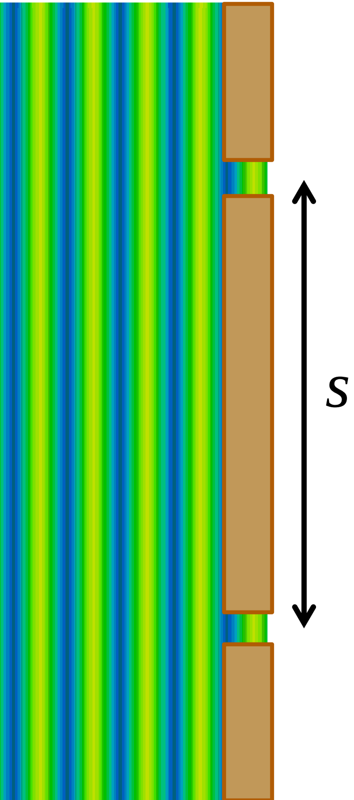
Young’s slits

An opaque mask has two slits cut in it, a distance s apart



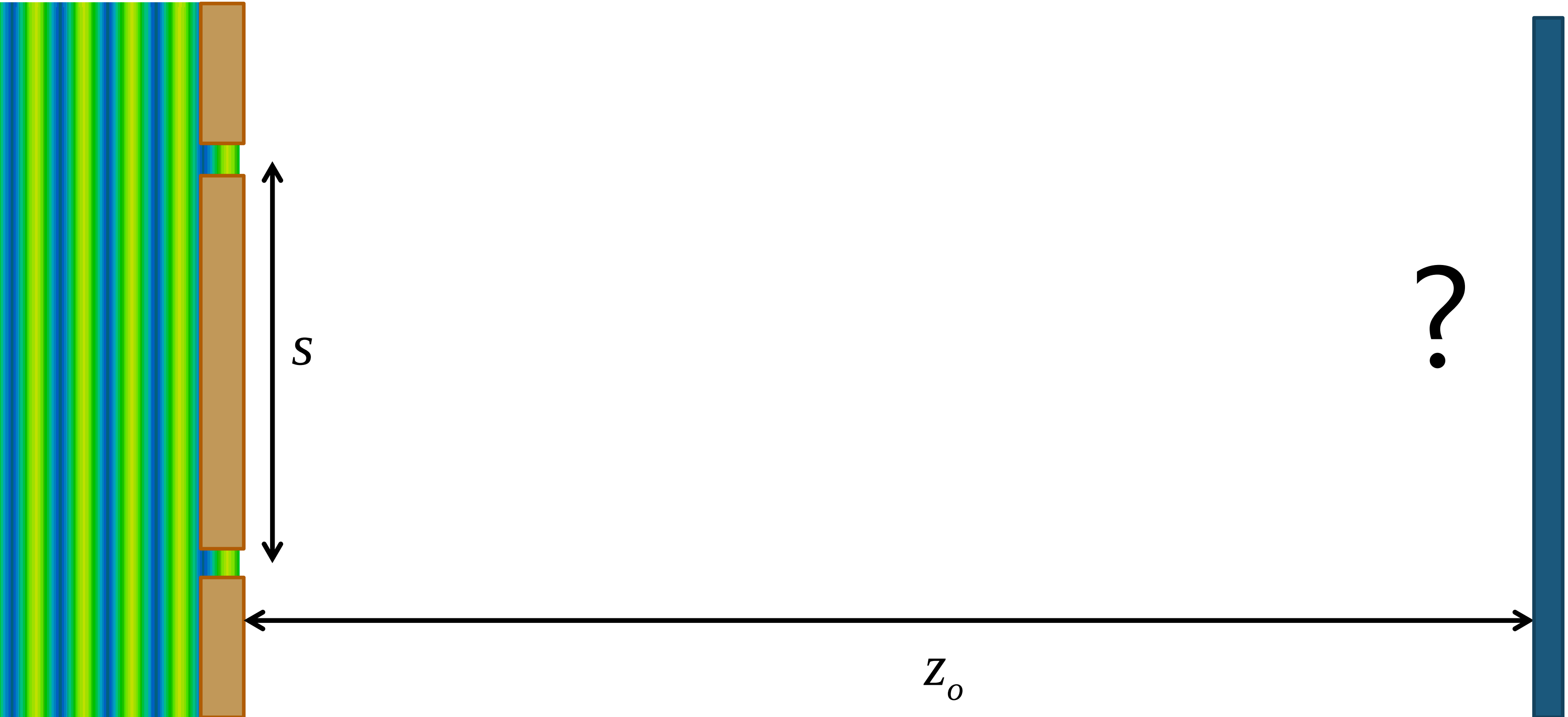
Young's slits

We shine a plane wave on the mask from the left



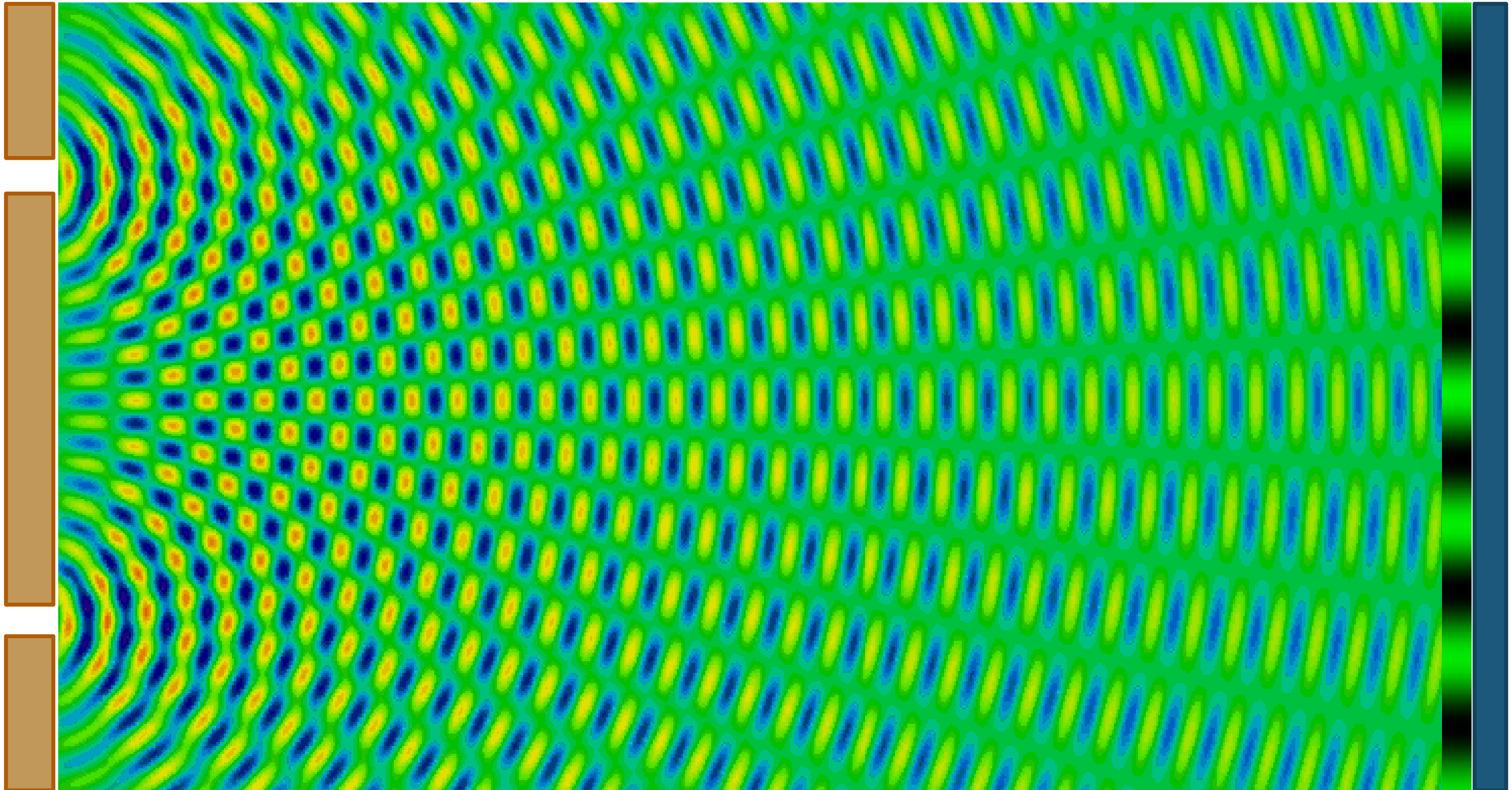
Young's slits

What will be the pattern on a screen at a large distance z_o?



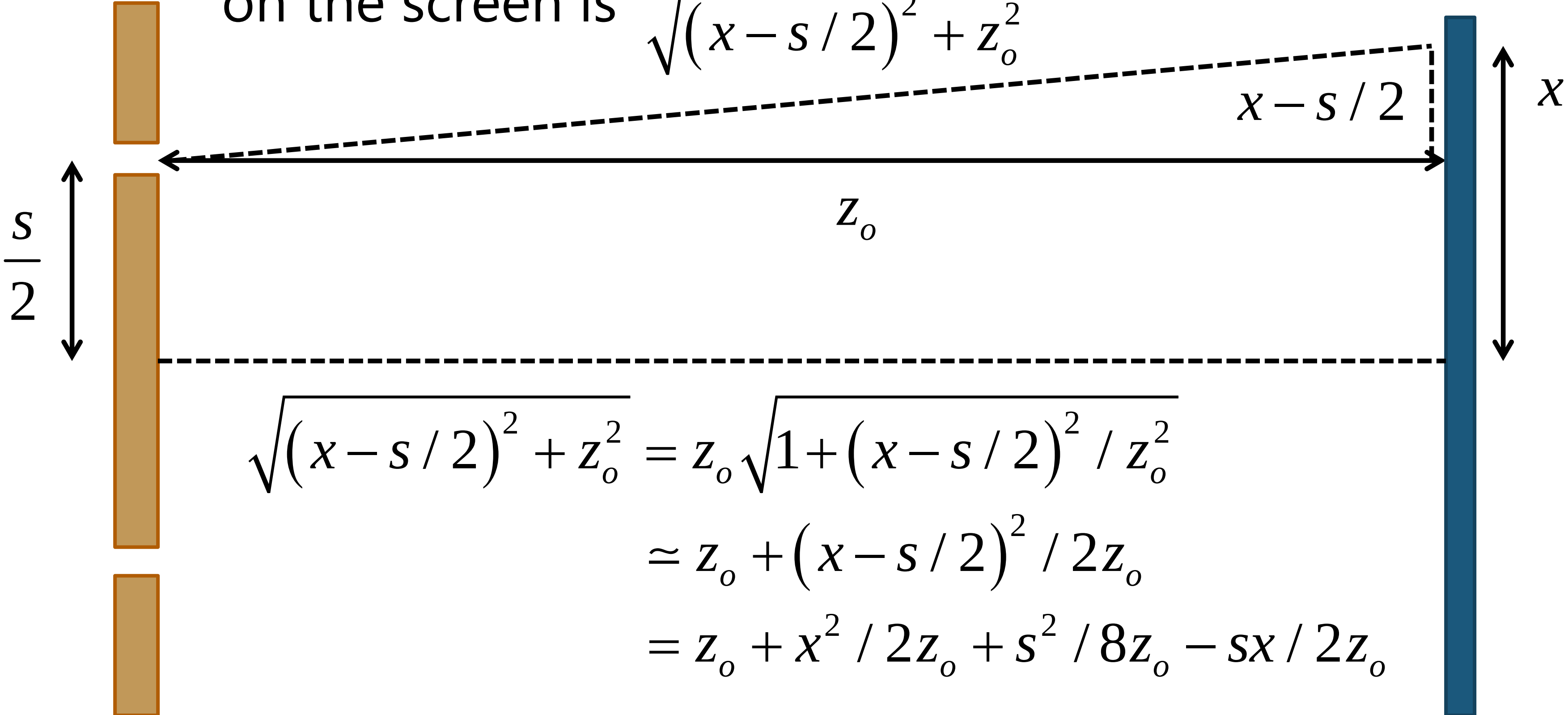
Young's slits

The slits as point sources give an interference pattern



Young's slits

The distance from the upper source to point x on the screen is $\sqrt{\left(x-s/2\right)^2+z_o^2}$



$$\sqrt{\left(x-s/2\right)^2+z_o^2}=z_o\sqrt{1+\left(x-s/2\right)^2/z_o^2}$$

$$\simeq z_o+\left(x-s/2\right)^2/2z_o$$

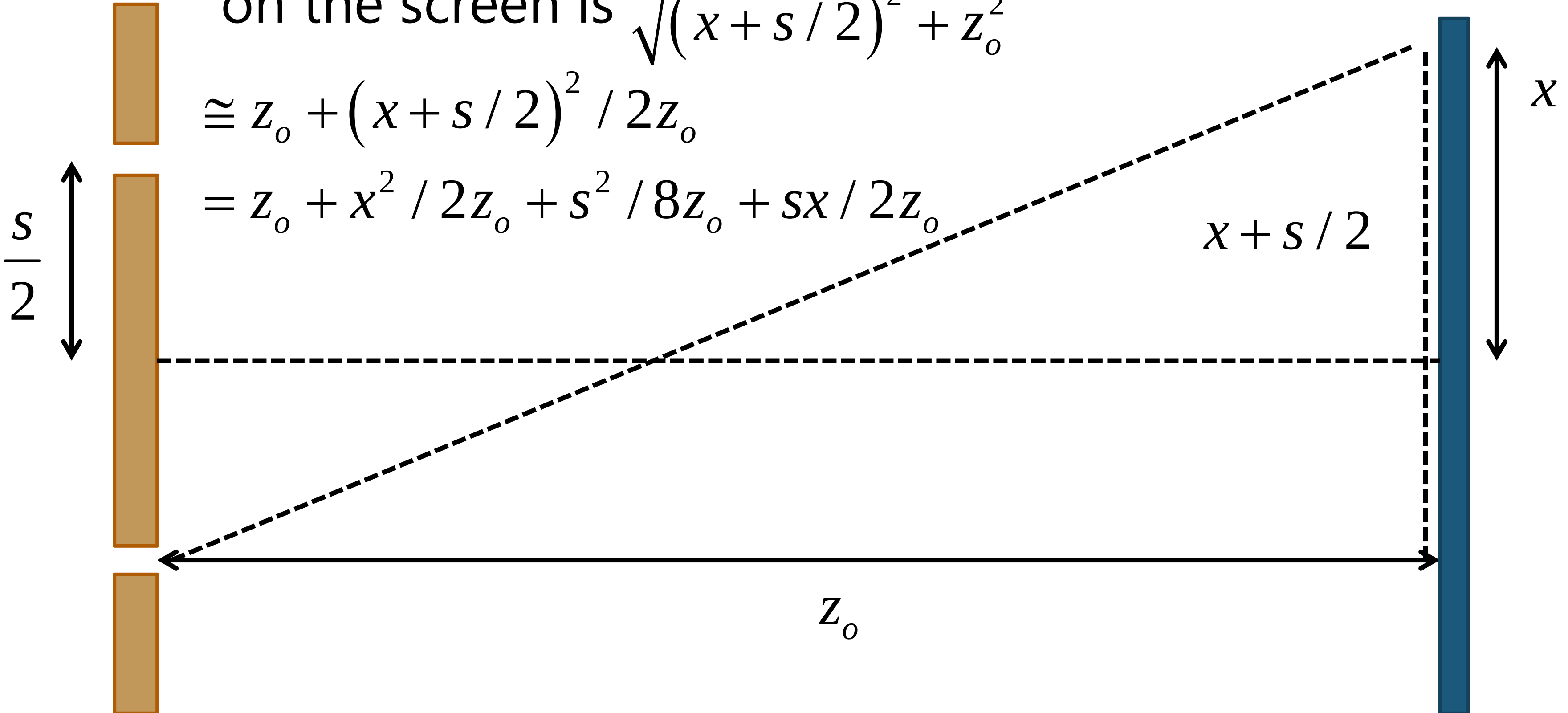
$$=z_o+x^2/2z_o+s^2/8z_o-sx/2z_o$$

Young's slits

The distance from the lower source to point x on the screen is $\sqrt{(x+s/2)^2+z_o^2}$

$$\cong z_o + (x+s/2)^2/2z_o$$

$$= z_o + x^2/2z_o + s^2/8z_o + sx/2z_o$$



Young's slits

For large z_o the waves are approximately uniformly "bright"

i.e., using exponential waves for convenience

$$\psi_s(x) \propto \exp\left[ik\sqrt{(x-s/2)^2+z_o^2}\right]+\exp\left[ik\sqrt{(x+s/2)^2+z_o^2}\right]$$

Using our approximate formulas for the distances gives

$$\psi_s(x) \propto \exp(i\alpha)\left\{\exp\left[ik(sx/2z_o)\right]+\exp\left[-ik(sx/2z_o)\right]\right\}$$

where $\alpha = k\left(z_o + x^2/2z_o + s^2/8z_o\right)$

Young's slits

Now $\exp(i\theta)+\exp(-i\theta)=2\cos(\theta)$

so $\psi_s(x)\propto\exp(i\alpha)\left[\exp\left(ik\frac{sx}{2z_o}\right)+\exp\left(-ik\frac{sx}{2z_o}\right)\right]$

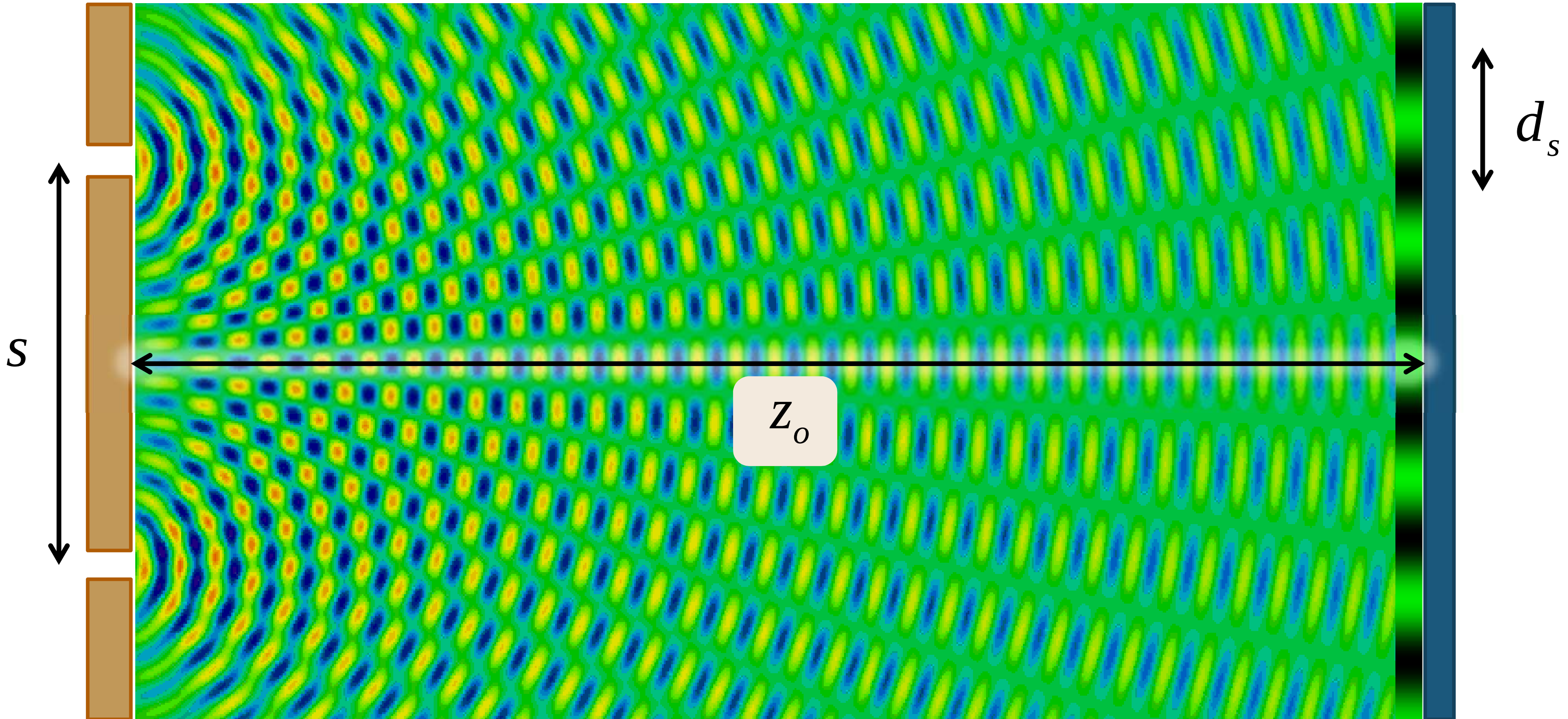
$$\propto\exp(i\alpha)\cos\left(k\frac{sx}{2z_o}\right)=\exp(i\alpha)\cos\left(\frac{\pi sx}{\lambda z_o}\right)$$

so the "intensity" of the beam is of the form

$$|\psi_s(x)|^2\propto\cos^2(\pi sx/\lambda z_o)=\frac{1}{2}\left[1+\cos(2\pi sx/\lambda z_o)\right]$$

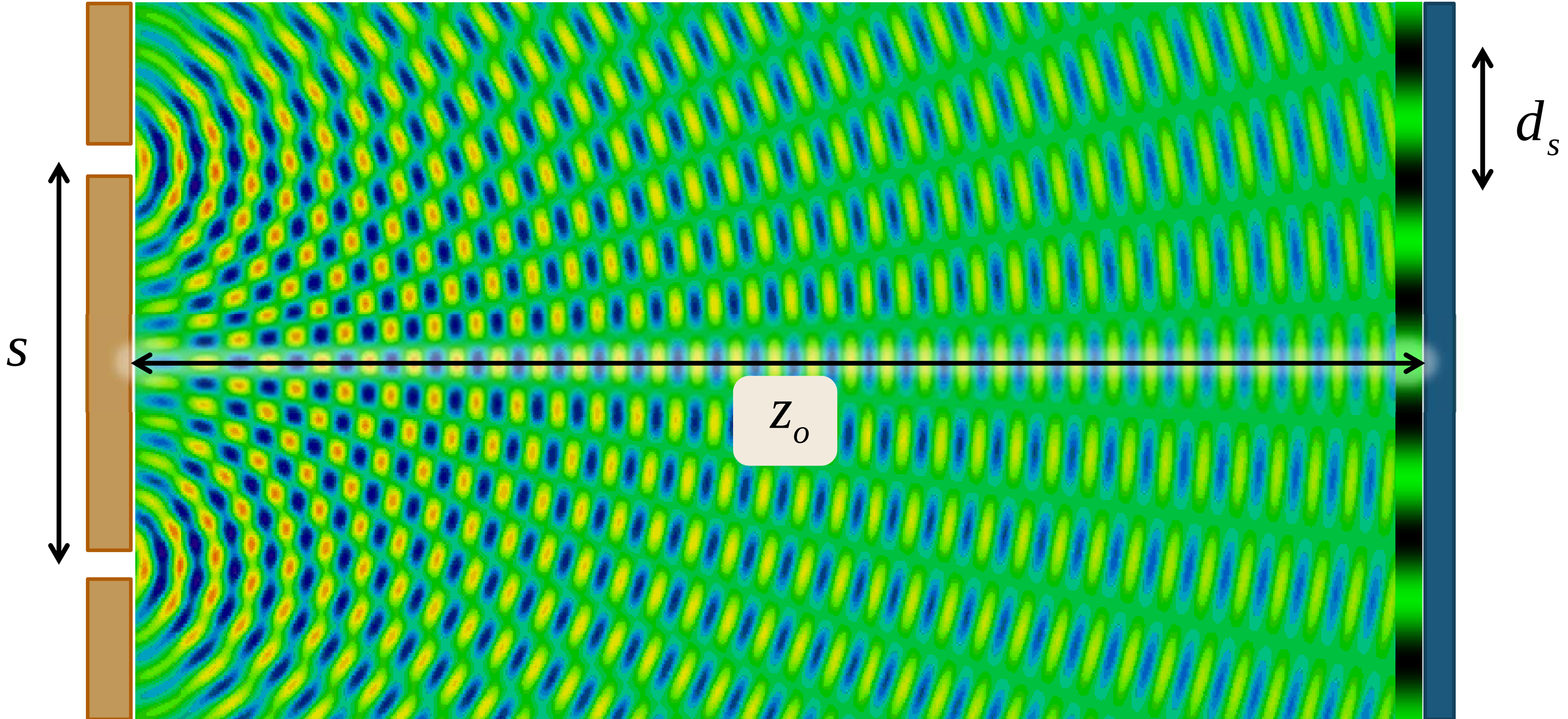
Young's slits

The interference fringes are spaced by $d_s = \lambda z_o / s$



Young's slits

This allows us to measure small wavelengths $\lambda = d_s s / z_o$



The quantum view of the world 3

Young's slits and quantum mechanics

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Young’s slits and quantum mechanics

- Young’s slits are easy to understand for light as waves
- If we think of light in terms of photons
  - we start to have more conceptual difficulty
    - and similarly for an electron wave incident on two slits

Young's slits and quantum mechanics

We can also do an experiment like this with electrons

- We can arrange to have an approximately "monochromatic" electron beam by
  - carefully pulling electrons out of a metal in a vacuum with a voltage
    - and we can effectively make them incident on these slits
      - in a good enough approximation to a plane wave

Young's slits and quantum mechanics

The typical conceptual problem we now have is that

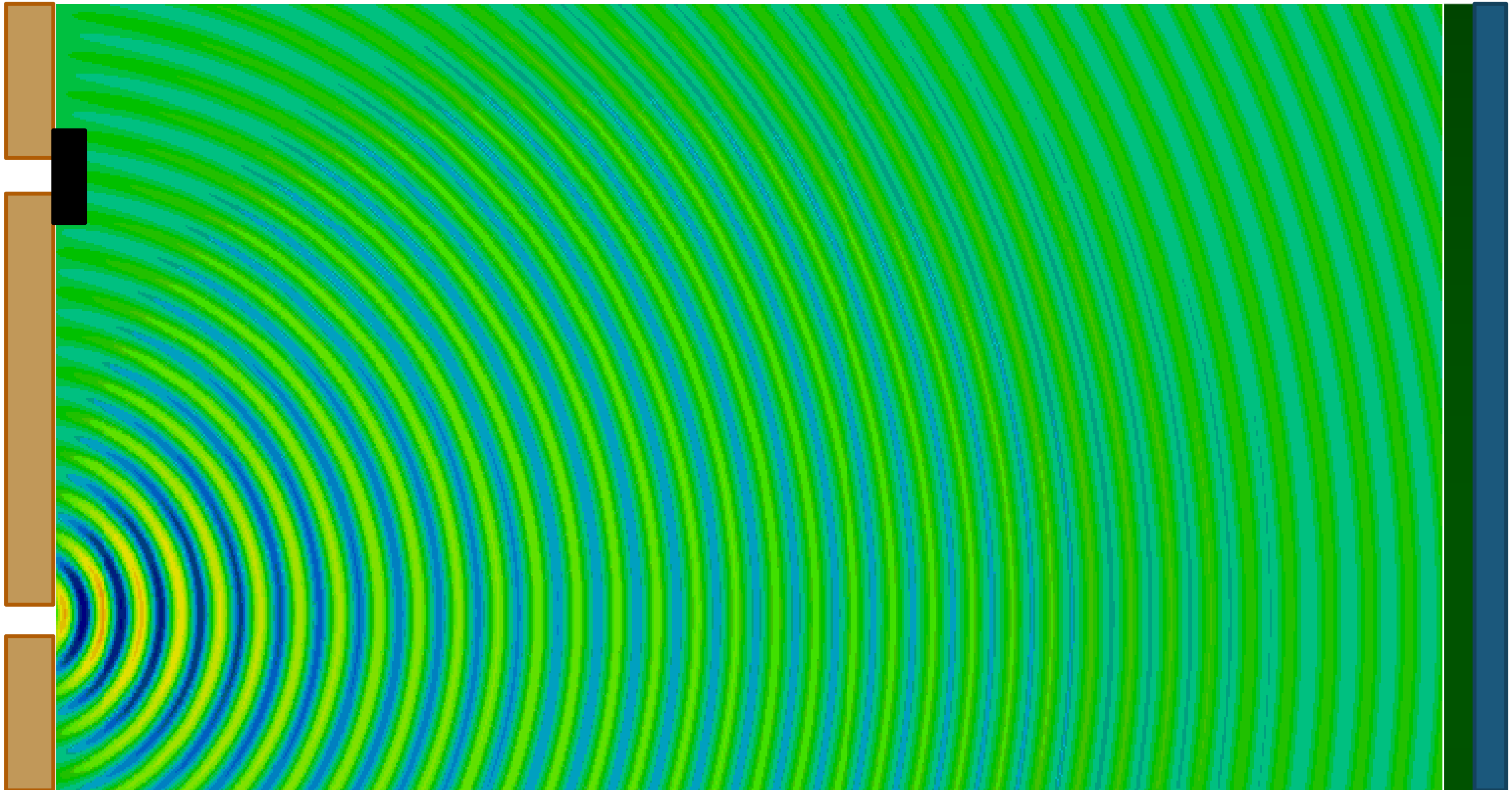
- our classical intuition tells us that
  - a particle has to go through one slit or the other
    - Surely a particle cannot go through two slits!

But

- if we insist the particle goes through one particular slit
  - we actually get no interference pattern

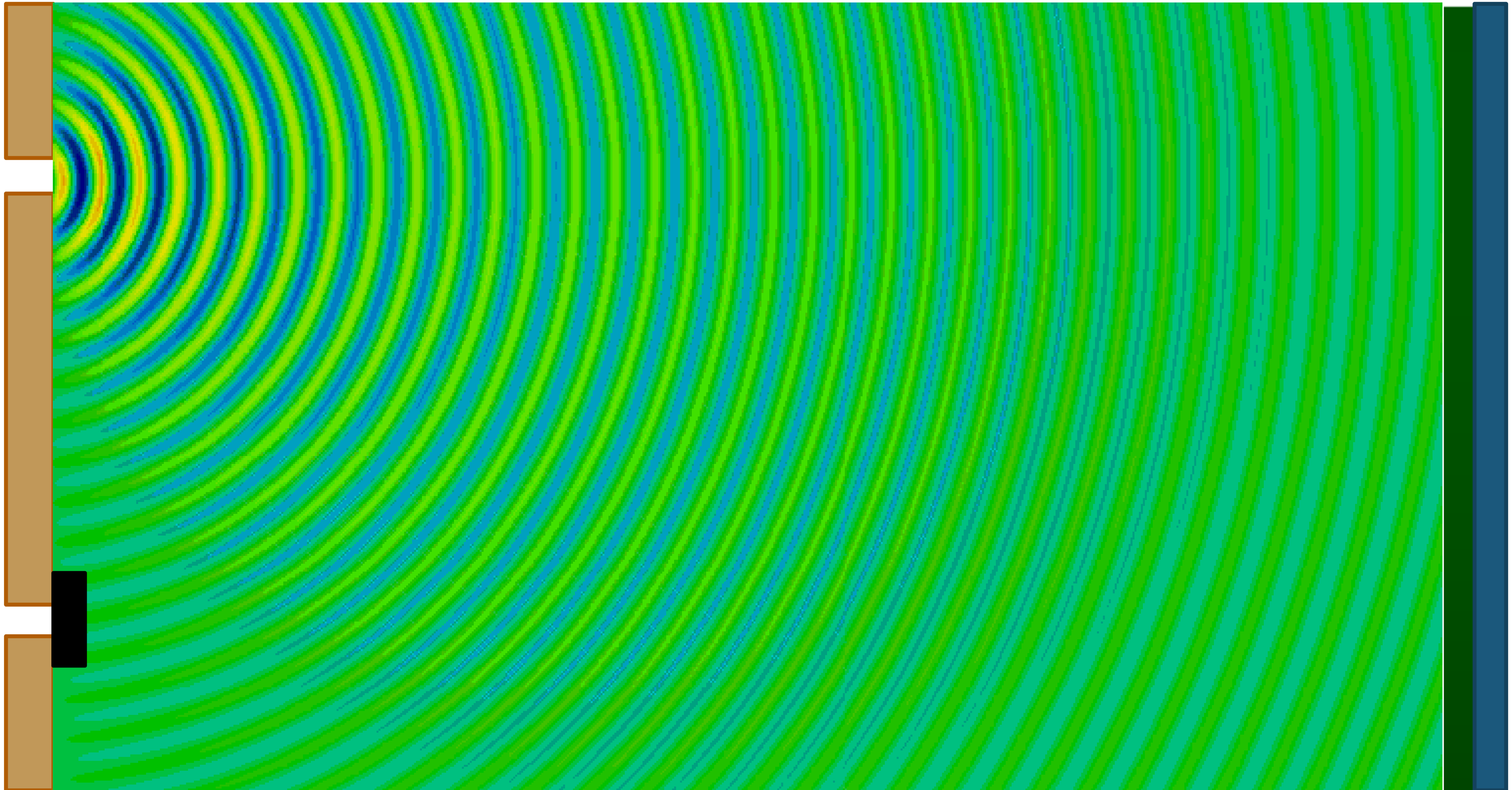
Young's slits

If the upper slit is blocked – no interference pattern



Young's slits

If the lower slit is blocked – no interference pattern



Young's slits and quantum mechanics

We might try to say that somehow

the interference pattern results

from the interference of

some particles going through the lower slit and

some through the upper slit

Young's slits and quantum mechanics

But, if we set this experiment up so we are sure

- there is only one photon or one electron in the apparatus at a time
  - we still build up the interference pattern!

For each electron or photon in the apparatus

- we get a "dot" on the screen
  - the pattern of dots builds up to give us the interference pattern!

Electron "two slit" experiment

An actual "two slit" experiment with electrons

- with electrons "landing" one by one
  - to give dots on the screen
    - still builds up an interference pattern
      - even though there is only one electron in the apparatus at a time

Web page

- https://www.hitachi.com/rd/research/materials/quantum/doubleslit/index.html

Video link

- https://www.hitachi.com/rd/research/materials/quantum/movie/video2a.html

Young's slits and quantum mechanics

How do we explain this?

- We say that
  - the electron hitting the screen
    - and exciting the phosphor to give off light
      - is a measurement of position

When we make a measurement of some quantity in quantum mechanics

- we collapse the system into an eigenstate of the quantity being measured
  - here position
- with probabilities given by Born's rule

This is the "collapse of the wavefunction"

It is arguable that we do not understand this

- but we do it anyway
  - because it works

"Shut up and calculate"