Variational optical processors

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Abstract: We introduce "variational optical processors," self-configuring photonic networks that learn modal representations of partially coherent or quantum optical fields through optimization, applicable to diverse classical and quantum optical processing tasks.

Classical and quantum light fields can be described in terms of modes. Modal decompositions play a central role in our understanding of multimode light field generation, propagation, and measurement. Some important examples include the decomposition of partially coherent light into so-called "natural modes" (a form of Karhunen-Loeve expansion [1]), coherency and density matrix eigendecomposition [1, 2], Schmidt decomposition of pure quantum bipartite states [2], and the Bloch-Messiah decomposition of multimode Gaussian states [3]. Methods relying on tomographic projection of the fields onto an *ad hoc* basis, usually requiring prior knowledge of the modes' geometry, have been realized to perform modal decomposition of multimode quantum and classical fields [4, 5].

Networks of Mach-Zehnder interferometers (MZIs) have proven highly effective in manipulating [6] and measuring [7] coherent multimode light. We now propose that these self-configuring meshes can be used to measure, process, and generate multimode light fields that exhibit some degree of partial (classical or quantum) coherence.

Here, we introduce a general framework for modal decomposition of partially coherent or quantum multimode light fields using a new class of optical devices coined "variational optical processors (VOP)." These networks consist of self-configuring MZI arrays whose parameters are optimized sequentially to e.g., maximize a function of the detected output signal. We find that in various cases – including Karhunen-Loeve expansions of classical and quantum light, Schmidt decomposition of bipartite states, and Bloch-Messiah decomposition of multimode squeezed vacua – the modal decomposition can be mapped to a sequential output optimization by variational principles. Once trained, the resulting device physically separates the input light fields into mutually orthogonal components that have the properties of the modal decomposition of interest (e.g., mutually incoherent, separable quantum states, supermodes, etc.). Furthermore, by sequentially learning the modes associated with the largest eigenvalues or singular values, our framework achieves favorable scaling in both physical hardware and required measurements compared to traditional tomographic techniques. Our method therefore paves the way to full characterization of multimode classical and quantum light fields in the various use cases shown in Fig. 1.

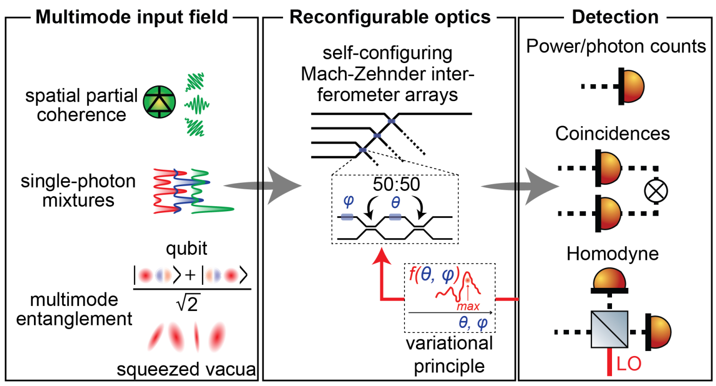

Fig. 1: **Variational optical processors.** Variational optical processors (VOPs) consist of a reconfigurable optical network (center) whose parameters (θ, ϕ) are trained through sequential optimization of a detected output (left). Variational principles map this optimization to a modal decomposition of the input light field.

The central element of the VOP is a reconfigurable optics consisting of cascaded self-configuring MZI layers, each parametrized by (θ_k, ϕ_k) which are vectors of phase parameters for layer k. The input multimode field can generally be described as a vector x, which may represent the amplitude of a light field over its spatial degrees of freedom, the wavefunction of a single photon, or the quadratures of a multimode Gaussian field. The output of the network y is a linear function of x parametrized by (θ_k, ϕ_k). We denote the signal detection function as $f(y)$. The key idea of VOP is that, for a given modal representation of the input light field Mode_k (Karhunen-Loeve, Schmidt, etc.), one can find a function f such that the following variational formulation holds:

$$\text{Mode}_k = \underset{\theta_k, \phi_k}{\text{argmax}}\ f(y|\theta_k, \phi_k). \tag{1}$$

We explain Eq. (1) with some specific examples. For instance, it was recently shown that for spatially partially coherent light, if $f(y) = \langle |y_k|^2 \rangle$ (the average power at the output port of the self-configuring layer k), Mode_k is the k-th natural mode of the field's Karhunen-Loeve expansion [8] (an observation that also holds for incoherent mixtures of delocalized single photons). Another example of importance in quantum optics is, if the VOP consist of two separate self-configuring networks, each receiving a single photon from a pure entangled state, and f is the average coincidence measured between ports k of the two networks, Mode_k corresponds to the k-th Schmidt mode [9]. We envision that more complex detection functions (homodyne detection, number basis or time bin measurements, etc.) may map to other representations of the input light field. In all of the above mentioned examples, the variational optimization is performed sequentially (layer by layer), such that each mode corresponds to the parameters of a single self-configuring layer.

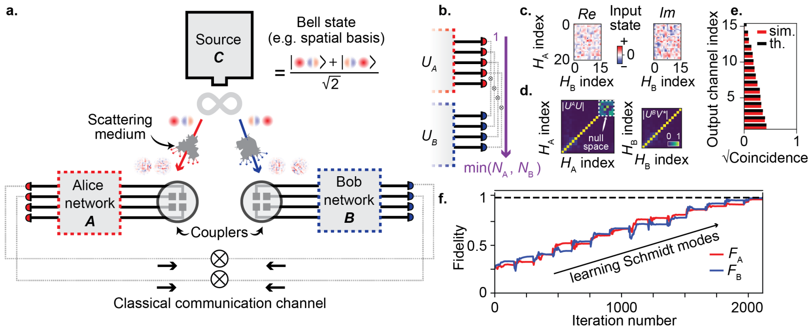

Fig. 2: **Applications of VOPs to quantum teleportation protocol. a.** Proposed setup for quantum teleportation through scattering media with VOPs. A source C is generating Bell states sent to Alice (A) and Bob (B) who analyze their single photon with their own VOP. A classical communication channel between the two allows them to perform coincidence measurements. **b.** Coincidence counts optimization sequence: from 1 to $\min(N_A, N_B)$ (on pairs of outputs ports from networks A and B). **c.** Random matrix input example (e.g., due to random scattering). **d.** Resulting eigenvector reconstruction from coincidence optimization. **e.** Resulting simulated square roots of coincidence counts (sim.) and singular values (th.). **f.** Fidelity of Schmidt modes in A (red) and in B (blue) over iteration number.

We now elaborate on potential applications of VOPs for entanglement distribution enabled by automatic Schmidt decomposition. By deploying VOPs at both ends of a communication channel, Alice and Bob can automatically perform a Schmidt decomposition of the entangled photons they receive, even after traversing complex scattering environments (see Fig. 2). This enables them to identify the optimal basis for quantum teleportation without prior knowledge of the scattering medium. Our protocol leverages sequential coincidence optimization within the VOPs, facilitating efficient entanglement distribution and robust quantum communication through scattering channels. To validate our method, we conducted simulations using a random matrix to represent the input state in Fig. 2**b-f**. By applying sequential coincidence maximization with VOPs, we demonstrated that the devices accurately identify the Schmidt modes (Fig. 2**d**) and corresponding Schmidt values (Fig. 2**e**), by effectively maximizing coincidence counts. The fidelity between the reconstructed and original states approaches unity (Fig. 2**f**), indicating that VOPs can reliably perform modal analysis of entangled states. These results highlight the potential of VOPs as tools for high-dimensional entanglement characterization and manipulation in practical quantum communication systems.

In conclusion, we have shown that self-configuring photonic networks, such as triangular arrays of MZIs, can automatically learn and measure multimode partially coherent or quantum light fields by sequentially optimizing their parameters to perform modal decomposition.

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